

## A neuro-Beta-elliptic model for handwriting generation movements

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**Abstract**— A neural network model for handwritten script generation is proposed, in which curvilinear velocity signals are approximated by the Beta profiles. For each Beta profile we associate an elliptic arc to fit the initial stroke in the trajectory domain. The network architecture consists of an input layer which uploads the set of Beta-elliptic characteristics as input, hidden layers and the output layer where script coordinates  $X(t)$  and  $Y(t)$  are estimated. A separate timing network prepares the input data. This latter involves the time-index starting time of each simple stroke for an appropriate handwriting movement signal. The experiments showed that the neural network model could be applied for the case of Latin handwriting scripts as well as Arabic handwriting scripts. New ways are proposed for the application of the neural network model such as: generation of complex handwriting movements, shape and character recognition.

**Keywords**- *handwriting generation; Beta-elliptic model; neural network model.*

### I. INTRODUCTION

Writing is a motor activity among the most complex and fastest engine of our directory. It seeks the coordinated activity of multiple muscles and joints of the upper limb to produce a series of graphic forms in a timely and precise enough to be recognized [1]. Despite the structural and functional complexity, humans are able to produce graphic forms accurately and stable and to adapt their production according to the conditions under which they must do. As the set of human motor behavior, the problem is how individuals can control and coordinate all elements of the motor system (here graphomotor) to accomplish this motor task. In a system composed of a large number of elements, it is unlikely that our brain can control and regulate all components independently to produce a motor action [2][3]. We must find a solution that reduces to a few numbers of variables to control, leading to the production of a stable and flexible behavior. To date, two major theoretical approaches have attempted to identify the principles that underlie the production, control and adaptation of coordinated movements: traditional theories of motor control on the one hand, and theories of motor dynamics coordination on the other.

*Models of handwriting:* In the follow we cite some of the important handwriting models, particularly emphasizing on

those used to inspire the proposed handwriting model. Two general approaches of handwriting model have been adopted by searchers in the past [4]. The first one, called the “bottom-up” approach, refers to computational models which attempt to empirically regenerate handwriting features such as velocity and acceleration profiles. This models do not take in consideration the neuromotor processes derived by the handwriting processes [5][6]. The second approach of handwriting modeling based on psychologically descriptive models [7][8]. These “top-down” models usually summarize many issues such as, motor learning, movement memory, planning and sequencing, co-articulation and task complexity of strokes [9]. The present work is closer to the “top-down” category.

In Hollerbach [10], the oscillation theory of handwriting represents an important class of handwriting models, is based on the theory that stroke data can be resolved into certain oscillatory mechanisms by Fourier decomposition. The oscillation theory was developed by Hollerbach [10] who proposed an insightful handwriting generation model where the hand-pen system is represented by two orthogonal pairs of opposing springs acting on an inertial load. It was emphasized that the oscillatory natural movements of this system be similar to real handwriting segments.

For the delta\_lognormal model of Plamondon [11], the curvilinear velocity of an appropriate handwriting script is approximated by a set of delta\_lognormal profiles, assuming that the cumulative time delays are saved for different samples of the neuromuscular system acting in the production of a rapid movement.

In Schomaker model [2], a neural network model is proposed wherein a network of oscillators outputs horizontal and vertical pen tip coordinates  $x(t)$  and  $y(t)$ . Network training, performed using a variation of rule, leads to uncertain results: the model performance depended critically on network parameters. Despite of the lacks of the performance of the model, Schomaker’s work clearly explains certain questions related to any possible handwriting model. Consequently, the handwriting process and therefore its model must have four basic phases both in sequencing and shaping of handwriting [2]:

1. System configuration: this step is differently identified as motor programming, coordinative structure gearing, preparation, planning.

2. Start of pattern: after configuring the system for the task to perform, there must be a signal releasing the pattern at the right time.

3. Execution of Pattern: the time of this phase and events that are accomplished depend on parameters such as quantity of time elapsed, the distance from a spatial target position or force target value, or even the number of motor patterns generated.

4. End of pattern: this stage deals with the termination of the movement.

More recently, Kalverama [6] proposed a handwriting model consisting on resolving stroke data to their Fourier components. This mathematical operation is described basing on “central target pattern generation”. This model suffers from several drawbacks. As a handwritten stroke, corresponding to a real motor sequence, has a finite duration the dynamic system adopted for its generation must be properly initiated and terminated. Fourier decomposition supposes an oscillators set having an initial state, prepared with accurate phase-relationship. In presence of a huge oscillator’s network, the initial state preparation can be a research challenge in itself, to achieve an accurate stroke learning, acquisition and production. These issues are not addressed in [6] which assume a prepared initial state. Another drawback is that in [6] a separate network has to be trained for every stroke.

The goal of the Gangadhar model [9] [12] is to have a stable cadence in an oscillators network, and to resolve the stroke output under its Fourier components, in terms of oscillatory behavior of the oscillator’s network. The considered network for strokes generation has 3 layers: input layer, oscillatory layer, and output layer. Each neuron in the input layer represents a separate stroke. Under relaxing conditions, all inputs are in a ‘low’ (0) state. To generate a stroke, the corresponding input line is set to a ‘high’ (1) state, and maintained in that state during the stroke execution. The oscillatory layer is composed of some sublayers. Neurons belonging to the same sublayer, which are connected in a ring topology, have the same oscillation frequency. Output layer has two outputs representing horizontal and vertical velocities ( $U_x$  and  $U_y$ ) of the pen tip. Each neuron of the output layer is connected to all the oscillators in the oscillator layer. Events related to the above 3-layered network are managed by a timing network. In our view the Gangadhar model [9] has several drawback and ambiguity: first of all we can’t understand the nature of the input of the network. Secondly the preparation of the initial stat of the ring topology chosen it can be a challenge in itself. Finally Gangadhar use a simple mathematical operation to generate pen tip coordinates  $x(t)$  and  $y(t)$ .

The main contributions in this paper are: the use of the Beta-elliptic model for features extraction giving a full description in static and kinematic field for handwriting. Secondly is the generation of  $x(t)$  and  $y(t)$  unlike the Gangadhar model [9] that generates velocities ( $U_x$  and  $U_y$ ) and then with a mathematical operation it generates  $x(t)$  and  $y(t)$ .

The outline of the paper is as follows: in section 2, we present our model for handwriting generation which contains two steps. In the first part we present the Beta elliptic model for features extraction. In the second part a mechanism for preparing the initial state of the network, training, and validation procedures are described. In section 3, simulation results are presented.

## II. THE PROPOSED MODEL

The proposed model is interested to online handwriting and inspired from the Beta-elliptic model for the analysis of complex handwriting movements. In that case, a simple stroke is approximated by a Beta profile in the dynamic domain which corresponds in turn to an elliptic arc in the trajectory domain. In addition, a stroke executed from an arbitrary starting position is characterized by ten parameters. The first five parameters reflect the timing properties of the neuromuscular networks involved in generating the movement, whereas the last five parameters describe the geometric properties of the generated trace. As shown in Fig.1, these characteristics are used as input for the neural network to generate the initial script.

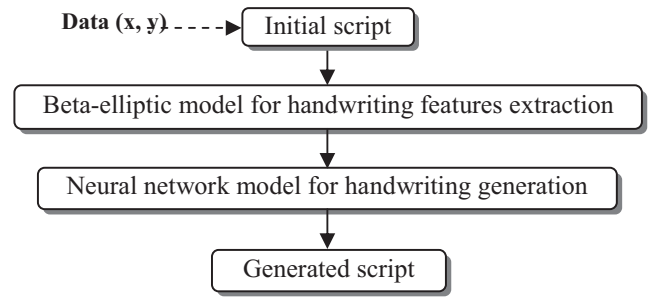


Figure 1. The architecture of the handwriting generation model.

### A. The Beta-elliptic model for handwriting features extraction

The beta-elliptic model is based on some assumptions: Firstly, it considers that handwriting movement, like any other highly skilled motor process, is partially programmed in advance. Secondly, it supposes that movements are represented and planned in the velocity domain since the most widely accepted invariant in movement generation is the beta function of the velocity profiles. In its simplest form, the model is based on a beta equation  $\beta(t, t_0, t_1, t_c, p, q)$  where  $t_0$  is the starting time,  $t_1$  is the ending time,  $p$  and  $q$  are intermediate parameters, as shown in (1). This equation describes the velocity profile in the kinematics domain which is in turn represented by an elliptic arc that characterizes the trajectory in the static domain [13] [14] [15] [16] [17] [18].

$$\beta(t, p, q, t_0, t_1) = \begin{cases} \left(\frac{t-t_0}{t_c-t_0}\right)^p \left(\frac{t_1-t}{t_1-t_c}\right)^q & \text{If } t \in ]t_0, t_1[ \\ 0 & \text{If not} \end{cases} \quad (1)$$

$p, q, t_0 < t_1 \in \mathbb{R}$

$$t_c = \frac{p * t_1 + q * t_0}{p + q} \quad (2)$$

The curvilinear velocity is given by (3).

$$V\sigma(t) = ((dx/dt)^2 + (dy/dt)^2)^{1/2} \quad (3)$$

The Beta-parameters ( $t_c$ ,  $\Delta = t_1 - t_c$ ,  $p$ ,  $q$ ,  $H$ : Beta amplitude) are presented in the Fig.2.

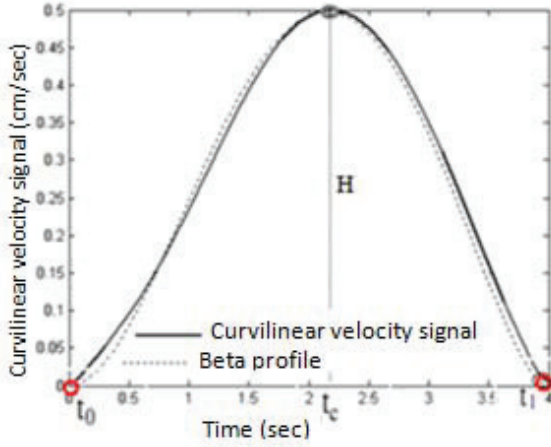


Figure 2. The different Beta-parameters.

The elliptic-parameters ( $x_0, y_0, a, b, \theta$ ) describe the static aspect of the handwriting movement;  $a$ : large axe of ellipse,  $b$ : small axe of ellipse, and  $x_0$  and  $y_0$  correspond to the coordinates of the ellipse centre  $O$  (Fig.3).

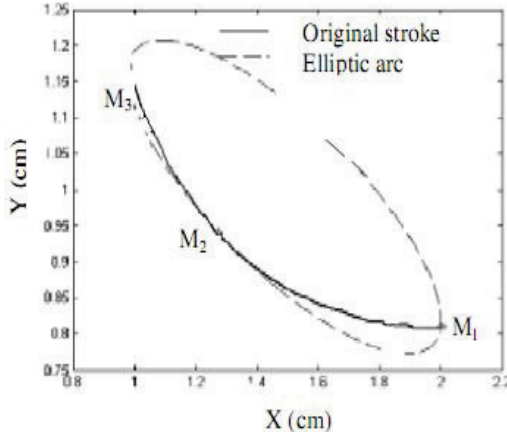


Figure 3. The different elliptic parameters.

The deviation angle  $\theta$  is formed by the ellipse and the horizontal axe, and obtained by the (4).

$$\theta = \arctang \left( \frac{(y_1 - y_0)}{(x_1 - x_0)} \right) \quad (4)$$

Basing on the Beta-elliptic model for handwriting features extraction, in the segmentation step each script is modeled in the dynamic domain by a series of Beta profiles and in the static domain by a series of elliptic arcs. These latter's represent the set of strokes composing the script.

To approximate a simple movement called stroke the Beta-elliptic model produce a set of ten parameters that describes the movement in both domains dynamic and static. Then for each elliptic arc we have ten parameters ( $t_c, p, q, t_0, t_1, a, b, x_0, y_0, \theta$ ). Unlike Gangadhar model these features will be used as input for our neural network for handwriting generation.

### B. The neural network architecture

The architecture of our network that learns to generate scripts has 3 layers: input layer, hidden layers and Output layer (Fig.4).

1) *Input layer*: The input layer contains 11 neurones, 10 neurones for the Beta-elliptic features (the static features given by elliptic equation and the dynamic features given by Beta equation). Also we use one neurone for the timing network.

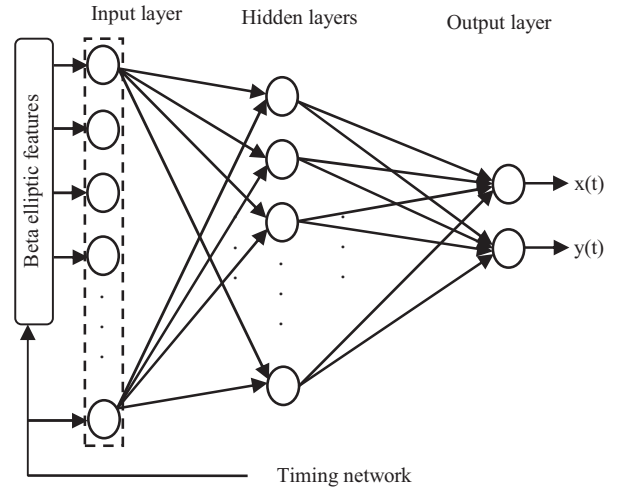


Figure 4. Neural network architecture.

The timing network acts as a synchronizer between the network input and output. In our model and in the extraction features stage every stroke was characterized with a set of beta profile and set of elliptic arc. Each pair (beta profile, elliptic arc) belongs to a time interval  $[t_0, t_1]$ . The timing network duplicates the features given by each couple (Beta profile, elliptic arc) in the corresponding time interval. Duplication is done so that each point  $(x(t), y(t))$  of the network output matches an input (the beta elliptic features) at time  $t$ .

2) *Hidden layers*: From table I it is clear that minimization of the training error corresponding to the learning algorithm is varied as a function of increasing the number of hidden layers and increasing the number of

neurons per hidden layer. From this table we have chosen network architecture with 4 hidden layers and for each layer we use 10 neurons.

TABLE I. MEAN ERROR TABLE

$Nh$	1	2	3	4
$Nn$				
5	6.2	$3.8 \cdot 10^{-1}$	$6.5 \cdot 10^{-2}$	$2.7 \cdot 10^{-3}$
10	$2.5 \cdot 10^{-2}$	$5.2 \cdot 10^{-3}$	$4.7 \cdot 10^{-4}$	$4.2 \cdot 10^{-5}$

$Nh$ : Number of hidden layers.

$Nn$ : number of neuron per hidden layer.

3) *Output layer*: This layer contains two neurons that represent the coordinates of pen tip motion  $(x(t), y(t))$ . The pen tip coordinates estimated by the network are expressed as weighted sum of the outputs of neurons in the hidden layer.

4) *Calculation of mean error*: The Backpropagation algorithm is used to train our network. The mean error shown in the Fig.5 is calculated using the (5).

$$E = \frac{1}{2} \sum_{i=1}^k \left\{ (D_x^i - x^i)^2 + (D_y^i - y^i)^2 \right\} \quad (5)$$

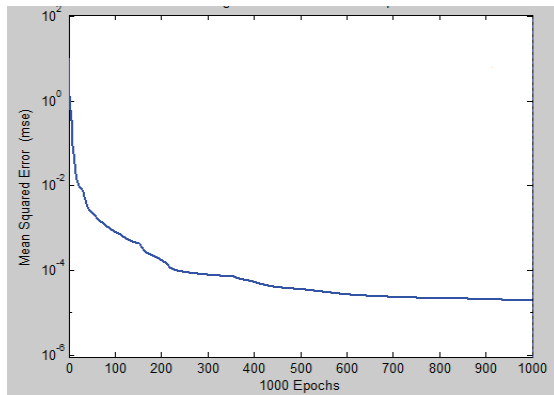


Figure 5. Training error corresponding to learning algorithm Plain Back propagation.

Where,  $D_x^i$  and  $x^i$  are the  $i$  th points in the desired and actual  $x(t)$  of the stroke respectively. Similarly subscript  $y^i$  indicates  $y(t)$ .  $E$  is the average reconstruction error in stroke;  $K$  is the number of points in the stroke.

### III. SIMULATION RESULTS

To test the performance of our model we use the ‘‘Mayastoroun’’ database [19]. This database contains a large variability of handwriting scripts, and written by many writers. It contains 5000 isolated digits, more than 13036 isolated lower and upper case letters, and 35017 isolated words from a 500 word lexicon (English, French and Arabic) are collected using digitizing tablet. To train our neural network we use 1000 scripts (digits, Latin and Arabic words and letters) for learning and 500 scripts for test. These scripts are represented by pen tip coordinates,  $x(t)$  and

$y(t)$ . Simulation results are given in the following figures (Fig.6, Fig.7, Fig.8, Fig.9 and Fig.10), where depicted respectively the initial scripts (a) and generated script (b) of the digit ‘‘9’’, the Latin letter ‘‘z’’, the Latin word ‘‘un’’, the Arabic word ‘‘حمل’’ and the Latin word ‘‘classe’’.

We remark good agreement between initial scripts and the scripts produced with the neural network for handwriting generation. In the applied domain, the potential of the present model to generate synthetic handwriting can probably be exploited as a generator of ‘‘handwritten CAPTCHAs’’ [20].

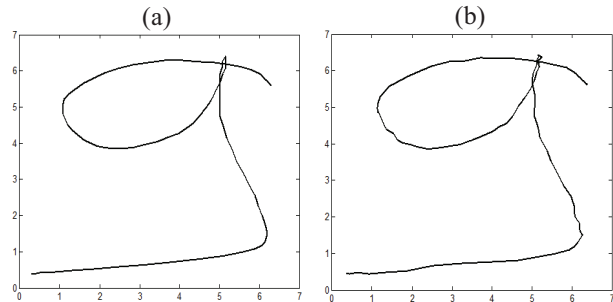


Figure 6. (a) the original scripts and (b) the generated scripts of the digit ‘‘9’’.

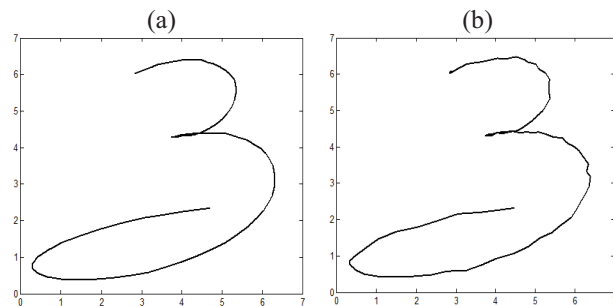


Figure 7. (a) the original scripts and (b) the generated scripts of the letter ‘‘z’’.

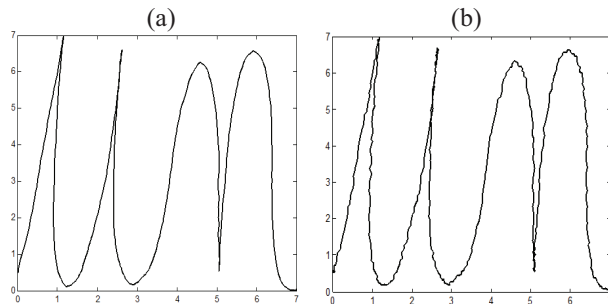


Figure 8. (a) the original scripts and (b) the generated scripts of the word ‘‘un’’.

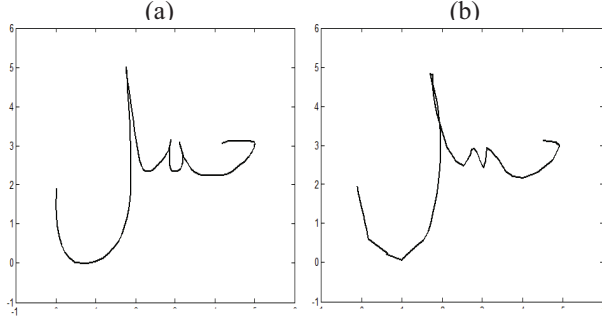


Figure 9. (a) the original scripts and (b) the generated scripts of the Arabic word 'حمل'.

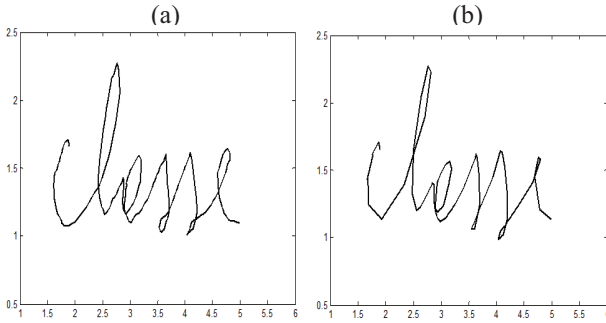


Figure 10. (a) the original scripts and (b) the generated scripts of the word 'classe'.

1) *Similarity degree measure*: The similarity degree between the original script and the generated script is measured with (6):

$$S(S_o, S_g) = 1 - \frac{\sqrt{\sum_{i=1}^n (x_{io} - x_{ig})^2 + (y_{io} - y_{ig})^2}}{2n} \quad (6)$$

Where:

$S(S_o, S_g)$  is the similarity degree between original script ( $S_o$ ) and generated script ( $S_g$ ).

$(x_{oi}, y_{oi})$  are the coordinates of the  $i$ th point in the original script.

$(x_{gi}, y_{gi})$  are the coordinates of the  $i$ th point in the generated script.

$n$  is the number of points of the original script.

We note that:

1.  $S(S_o, S_g) \in [0, 1]$
2.  $S(S_o, S_g) = 1 \Rightarrow S_o = S_g$
3.  $S(S_o, S_g) = \alpha \Rightarrow S_g$  (generated script) is similar with  $\alpha$  degree to the  $S_o$  (original script).

From table II we note that the generated scripts are similar with a degree of the order of 0,8 to the original scripts. We conclude that the neural network can generate scripts with acceptable performance. These results can be ameliorated (increase the similarity degree) by considering a more large learning data base.

TABLE II. THE MEASURE OF SIMILARITY

Scripts	The similarity degree
Digit '9'	0.8231
Letter 'z'	0.8342
Word 'un'	0.8104
Word 'حمل'	0.8067
Word 'classe'	0.8020

#### IV. CONCLUSION

In this paper we present a neural network model of handwritten script generation in which script velocity is expressed as a Beta-elliptic characteristics. This characteristics was used as input for the neural network to generate the script pen tip coordinates,  $x(t)$  and  $y(t)$ . Our proposed model is successfully tested on the considered database formed with Latin and Arabic letters and digits. To this end, the present model has to be trained on a large database of online cursive data. The model can also be trained on data from a specific individual.

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