Online Signature Verification Based on Legendre Series Representation. Robustness Assessment of Different Feature Combinations

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***Abstract*—In this paper, orthogonal polynomials series are used to approximate the time functions associated to the signatures. The coefficients in these series expansions, computed resorting to least squares estimation techniques, are then used as features to model the signatures. Different combinations of several time functions (pen coordinates, incremental variation of pen coordinates and pen pressure), related to the signing process, are analyzed in this paper for two different signature styles, namely, Western signatures and Chinese signatures of a publicly available Signature Database. Two state-of-the-art classification methods, namely, Support Vector Machines and Random Forests are used in the verification experiments. The proposed online signature verification system delivers error rates comparable to results reported over the same signature datasets in a previous signature verification competition.**

***Keywords*-Online Signature Verification; Legendre polynomials approximations; Feature combinations.**

I. Introduction

Signature verification plays an important role in the field of personal authentication, being the most popular method for identity verification. Financial and administrative institutions recognize signatures as a legal means of verifying an individual's identity. In addition, signature verification is a non-invasive biometric technique, and people are familiar with using the signatures for identity verification in their everyday life.

Different categories of signature verification systems can be distinguished, namely, offline and online systems [1]. In the offline case, only the image of the signature is available, while in the online version, dynamic information about the handwriting process is included. Online verification systems are gaining more and more interest since adding dynamic information makes signatures more difficult to forge. It can be expected that these systems would be more reliable than the offline ones. In addition, electronic pen-input devices are gaining popularity for signature acquisition in several daily applications, being the digitizer tablets and the PDAs the most popular ones.

In online verification systems the signature is parameterized by different discrete time functions, *e.g.*, pen coordinates, pen pressure and pen inclination angles. Researchers have long argued about the effectiveness of the different time functions available for online signature verification. During the First International Signature Verification Competition (SVC 2004), the results using only pen coordinates outperformed those adding pen pressure and pen inclination angles [2]. Since then, several works have been presented concerning the most suitable set of parameters for modeling the signatures. In [3], the authors assert that using pen coordinates leads to better performance than using pen pressure in addition to pen coordinates. In [4], several parameters are compared and the authors conclude that pen coordinates and some derived parameters are the most reliable. Even the time variability between training and testing data acquisition sessions was considered in [5], where the authors conclude that pen pressure is the most unreliable parameter, and pen inclination angles are too unstable (as shown by most of the researchers), being pen coordinates the most robust time functions in the presence of a long term time variability. On the other hand, some works showed an improved performance when combining the pen coordinates information with the pen pressure and pen inclination angles [6]. The conflicting results observed in the literature make the discussion still open. Moreover, most of the works do not consider the cultural origin of the signatures.

In this paper, orthogonal polynomials series are used to approximate the time functions associated to the signatures. The coefficients in these series expansions are then estimated and used as features to model the signatures. Different combinations of several time functions (pen coordinates, incremental variation of pen coordinates and pen pressure), related to the signing process, are analyzed for two different signature styles, namely, Western signatures and Chinese signatures. Two state-of-the-art classification methods, namely, Support Vector Machines (SVMs) and Random Forests (RFs) are then used in the verification experiments and the performance of these methods is evaluated.

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The main contributions of this paper are the following:

- A new feature extraction approach based on orthogonal polynomials series expansion of the time functions associated to the signing process is proposed. To the best of the authors' knowledge this is the first time that this approach is used in the context of signature verification.
- Different combinations of the time functions associated to the signing process are studied and the pros and cons of the different combinations are analyzed.
  The experiments are performed on the most recent signature datasets, containing Western signatures and Chinese signatures, which have been used in the latest signature verification competition. For the results, the EER (Equal Error Rate) and the cost of the log-likelihood ratios $\hat{C}_{llr}$ are reported.

The paper is organized as follows. The feature extraction approach is described in Section II. In Section III the database is described. Section IV is devoted to the description of the experiments and in Section V the results are presented and discussed. Finally, some concluding remarks are given in Section VI.

II. Feature extraction

Several methods have been proposed in the literature for online signature verification. The methods differ basically in the way they perform the feature extraction and in the classification approach they employ. The different features can be classified into local features, calculated for each point in the time sequence, and global features, calculated from the whole signature. Many researchers accept that approaches based on local features achieve better performance than the ones based on global features, but still there are others who favor the use of global features [7], [8].

When using global features, feature vectors have a fixed amount of components regardless the signature length. This represents an advantage since it makes the comparison between two signatures easier with respect to the case of having different feature vector lengths. Moreover, a fixed-length model of the signatures can be required for certain biometric applications ([9], [10]). In [11], a fixed-length representation of the signatures is proposed based on the Fast Fourier Transform (FFT).

In this paper, a fixed-length representation of the signature is proposed based on the approximation of the different time functions by orthogonal polynomials.

A. Orthogonal polynomials series expansions

A family of functions $\{g_i\}$ in (in general) an infinite dimensional functional space $H([a,b])$, defined in the domain $[a,b]$, is said to be orthonormal with respect to an inner product $\langle\cdot,\cdot\rangle$ in $H([a,b])$ if $\langle g_i, g_j\rangle = \delta_{ij}$, where δ_{ij} is the Kronecker delta.

Provided the inner product space $H([a,b])$ is complete with respect to the metric induced by the inner product, a set of orthonormal basis functions $\{h_i\}_{i=0}^{\infty}$ can be defined. In this case, any function $f \in H([a,b])$ can be uniquely represented by a series expansion in the orthonormal basis, that is

$$f = \sum_{i=0}^{\infty} \alpha_i h_i, \tag{1}$$

where

$$\alpha_i = \langle f, h_i\rangle. \tag{2}$$

It is not difficult to prove that the best (in the sense of the metric induced by the inner product) approximation of $f \in H([a,b])$ in an N-dimensional subspace is given by

$$f \approx \sum_{i=0}^{N} \alpha_i h_i. \tag{3}$$

B. Time function approximation by Legendre orthogonal polynomials

The idea here is to approximate the time functions measured during the signature acquisition stage by a finite series expansion in orthonormal polynomials in the interval $[0,1]$, and to use the series expansion coefficients as features. Particularly, Legendre polynomials are considered in this paper. In this case, the approximation equation (3) becomes

$$f(t) \approx \sum_{i=0}^{N} \alpha_i L_i(t), \tag{4}$$

where $L_i(t)$ are the normalized orthonormal Legendre polynomials[1] in the interval $[0,1]$.

Since the time functions $f(t)$ are unknown, the coefficients in the truncated series expansions (4) cannot be computed as in (2) but rather they have to be estimated from a set of M (usually larger than $N+1$) samples of the function at the time instants $\{t_1, t_2, \cdots, t_M\}$.

In matrix form, equation (4) at the time instants $\{t_1, t_2, \cdots, t_M\}$ can be written as

$$\underbrace{\begin{bmatrix} f(t_1) \\ f(t_2) \\ \vdots \\ f(t_M) \end{bmatrix}}_{\mathbf{f}} = \underbrace{\begin{bmatrix} L_0(t_1) & L_1(t_1) & \cdots & L_N(t_1) \\ L_0(t_2) & L_1(t_2) & \cdots & L_N(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ L_0(t_M) & L_1(t_M) & \cdots & L_N(t_M) \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_N \end{bmatrix}}_{\alpha} \tag{5}$$

It is well known that the solution $\widehat{\alpha}$, in the least squares sense, of the overdeterminated system of equations (5) is

[1]The polynomials are orthonormal with respect to the standard inner product

$$\langle h_i(t), h_j(t)\rangle = \int_0^1 h_i(\tau) h_j(\tau) d\tau.$$

given by $\widehat{\alpha} = \mathbf{L}^{\dagger}\mathbf{f}$, where $\mathbf{L}^{\dagger} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T$, stands for the left pseudo-inverse of $\mathbf{L}$.

To illustrate the above estimation procedure, the x and y pen coordinates associated to a signature, and the corresponding approximations using Legendre polynomials with orders $N = 21$, $N = 15$ and $N = 10$, are shown in Fig. 1. The Best FIT[2] between the measured and the approximated time functions, for the above mentioned Legendre polynomial orders, are given in Table I.

Table I
BEST FIT BETWEEN THE MEASURED AND THE APPROXIMATED TIME FUNCTIONS.

N	$FIT_x[\%]$	$FIT_y[\%]$
21	77.7955	70.7341
15	68.9708	62.9579
10	57.6664	53.3995

Experimental results showed that further increasing the polynomial orders does not substantially improve the approximation accuracy. This is an expected result, taking into account the bias-variance tradeoff inherent to least squares estimation from noisy data.

A similar approach to represent handwritten symbols but using function moments instead of the corresponding series expansions was presented in [12].

III. Signature database

The publicly available SigComp2011 Dataset [13] presented within ICDAR 2011 is used. This database has two separate data sets, one containing Western signatures (Dutch signatures) and the other one containing Chinese signatures. The data was collected from realistic and forensically relevant scenarios, in the sense that the signatures were acquired using a ballpoint pen on paper, which is the natural writing process. This is in contrast to the approach of other researchers who tested signatures produced on a PDA or with a Wacom-stylus on a glass or plastic surface.

The considered online data was collected with a WACOM Intuos3 A3 Wide USB Pen Tablet. Measured data consists of three discrete time functions: pen coordinates x and y, and pen pressure p. In addition to the raw data, the incremental variation of the x and y pen coordinates (Δx and Δy, respectively) are computed. In [3] and [4] several features are considered and it is argued that x,y, Δx and Δy are among the most reliable ones.

Each of the datasets in the SigComp2011 database is divided into two sets, namely, the Training Set and the Testing Set. The online Dutch dataset consists of: 10 authors with 240 genuine and 119 forged signatures for the Training Set and 54 authors with 1296 genuine and 611 forged signatures for the Testing Set. The online Chinese dataset consists of: 10 authors with 230 genuine and 429 forged signatures for the Training Set and 10 authors with 219 genuine and 461 forged signatures for the Testing Set.[3] The forgeries in the database are skilled forgeries. Skilled forgeries are simulated signatures in which forgers, who are different writers than the reference one, are allowed to practice the reference signature for as long as they deem it necessary.

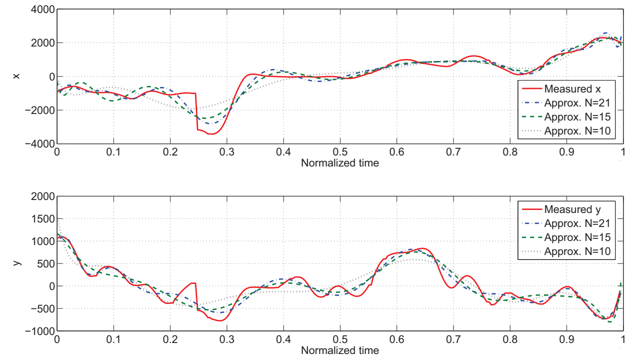

Figure 1. Time functions: x and y pen coordinates (red solid line) and their corresponding approximations by Legendre polynomials with orders $N = 21$ (blue dashed line), $N = 15$ (green dash-dotted line) and $N = 10$ (black dotted line).

IV. Evaluation protocol

Several combinations of time functions are considered to represent the signatures. Discussions about the robustness of the different time sequences used to model the signatures have been carried out in recent years. In spite of the conflicting results presented so far, most of the researchers agree that pen pressure has not as discriminative power as it was expected to have when using it alone. Experimental results confirming this fact have been also obtained by the authors, but they have not been included here due to space limitations. Based on the above comments, pen pressure is only used as a complementary time function in this paper. The following combinations of time functions will be considered to assess the verification performance:

- pen coordinates: x, y
- pen coordinates and pen pressure: x, y, p
- incremental variation of pen coordinates: Δx,Δy
- incremental variation of pen coordinates and pen pressure: Δx,$\Delta y, p$
- pen coordinates and incremental variation of pen coordinates: x, y,Δx,Δy

[2]The Best FIT is defined as

$$\text{Best FIT} = 100\left(1 - \frac{\|x - x_{approx}\|}{\|x - x_{mean}\|}\right).$$

[3]Amounts of genuine and forged signature samples may differ from those in [13] since when making signatures available for the research community some of them were missing [14].

- pen coordinates, incremental variation of pen coordinates and pen pressure: $x, y,\Delta x,\Delta y, p$.

For the sake of completeness, two well known state-of-the-art classifiers are used to perform the verification experiments, namely, Support Vector Machines [15] and Random Forests [16].

For each dataset, namely, Chinese and Dutch, the optimization of the meta-parameters of the system is performed over the corresponding Training Set while the corresponding Testing Set is used for independent testing purposes.

The tuning parameters to adjust are the order of the Legendre polynomials and the internal parameters of the classifiers. To select the most suitable order for the Legendre polynomials, tests varying this parameter from 1 to 25 were carried out. For the SVM classifier, the tuning parameters[4] are the scale σ^2 in the Radial Basis Functions (RBF) kernel[5], and the regularization parameter $C > 0$ providing a tradeoff between model complexity and the training error, in the SVM cost function. The linear and polynomial kernels were also tested, but the RBF gave the best results. For the RF classifier, there are basically two tuning parameters to adjust, namely, the number of trees to grow and the number of randomly selected splitting variables to be considered at each node. In general, the sensitivity to those parameters is not meaningful [18], and the default values are a good choice.

To obtain statistically significant results, a 5-fold cross-validation (5-fold CV) is performed over the Testing Set to estimate the testing errors. As it is usual, the signatures from each of the authors in the dataset are equally distributed among the 5 folds in the 5-fold CV. For each instance of the 5-fold CV, a signature model is trained for each writer in the dataset. Only genuine signatures are used for training purposes. When training a signature model for a particular writer, two classes are involved, namely, genuine and forged. For training the genuine class, the subset of genuine signatures of that writer available in the corresponding training set of the 5-fold CV is used, while the subset of genuine signatures of all the remaining writers in the dataset available in the corresponding training set of the 5-fold CV is used for training the forged class. For testing purposes, the subset of genuine and forged signatures of the writer under consideration available in the corresponding testing set of the 5-fold CV are used. Only skilled forgeries are considered to calculate the testing errors. Random forgeries are not considered for testing since they seldom appear in real situations.

[4]These parameters were optimized using the tune.svm routine of the e1071 Package [17], with values within the range 10^{-10} to 10^{10}.

[5]The RBF kernel is defined as

$$K(\mathbf{x}(n), \mathbf{x}(k)) = e^{\|\mathbf{x}(n)-\mathbf{x}(k)\|^2/\sigma^2}.$$

To evaluate the performance, the EER is calculated, using the Bosaris toolkit[6], from the Detection Error TradeOff (DET) Curve [19] as the point in the curve where the FRR (False Rejection Rate) equals the FAR (False Acceptance Rate). In addition, the cost of the log-likelihood ratios $\hat{C}_{llr}$ and its minimal possible value $\hat{C}_{llr}^{min}$ [20] are computed using the same toolkit. A smaller value of $\hat{C}_{llr}^{min}$ indicates a better performance of the system. The use of such measurements for evaluating the signature verification performance is proposed in [13]. In that work, the authors highlighted the importance of computing the likelihood ratios since they make Forensic Handwriting Experts (FHEs) able to combine the results obtained from an automatic verification system with other evidence presented in a court of law [21].

V. Results and discussion

Results for the Dutch and Chinese datasets are presented in Tables II and III, respectively. The results in Table II were obtained with the following values of the SVM tuning parameters: $\sigma^2 = 10^7$ and $C = 1$. On the other hand, the results in Table III were obtained with the following values of these tuning parameters: $\sigma^2 = 10^7$ and $C = 10$. For both datasets, the tuning parameters of the RF methods were set to: number of trees $= 500$, number of randomly selected splitting variables $= \sqrt{P}$, where P is the dimension of the feature vector. Further, for all the experiments, the order of the Legendre polynomials was set to $N = 21$.

Analyzing the contribution of the pen pressure when combined with the other considered time functions, it can be observed that incorporating the pen pressure improves the obtained performance in most of the cases for Dutch and Chinese data independently of the classifier being used. This observation agrees with the ideas presented in [6] where authors stated that incorporating pen pressure could improved the obtained results depending on the classification algorithm. Another important observation is that, in almost all the cases, using the computed sequences Δx and Δy leads to a better performance than using directly the measured sequences x and y. The Δx and Δy sequences are calculated as the difference between neighboring points in the x and y time sequences, respectively, as suggested in [3]. These parameters can be interpreted as the speed in x and y [4] and, in the analyzed cases, show a bigger discriminative power than the x and y coordinates. In [4] the pen coordinates and the speed are listed among the most reliable features. Results obtained in the experiments make it possible to agree with that, highlighting the advantages of using the speed sequences over the position ones.

From Tables II and III, it can be noticed that the results obtained with RF are better than the ones obtained with SVM. To the best of the authors' knowledge, there are no conclusive results regarding which one, between RF and

[6]http://sites.google.com/site/bosaristoolkit/

Table II
RESULTS FOR THE DUTCH DATASET.

Features	Class.	EER	$\hat{C}_{llr}$	$\hat{C}_{llr}^{min}$
x, y	SVM	12.32	0.461	0.3832
	RF	8.94	0.3265	0.288
x, y, p	SVM	12.39	0.4322	0.373
	RF	7.39	0.2751	0.2396
Δx,Δy	SVM	8.55	0.3482	0.3032
	RF	5.92	0.2648	0.2035
Δx,$\Delta y, p$	SVM	7.63	0.3501	0.2657
	RF	**5.91**	**0.237**	**0.195**
x, y,Δx,Δy	SVM	12.35	0.461	0.3833
	RF	6.46	0.2497	0.2122
x, y,Δx,$\Delta y, p$	SVM	12.54	0.4478	0.3744
	RF	7.24	0.2676	0.251

System	Acc.	$\hat{C}_{llr}$	$\hat{C}_{llr}^{min}$
commercial	96.27	0.2589	0.1226
1st. non-commercial	93.49	0.4928	0.2375

Table III
RESULTS FOR THE CHINESE DATASET.

Features	Class.	EER	$\hat{C}_{llr}$	$\hat{C}_{llr}^{min}$
x, y	SVM	12.67	0.5222	0.4419
	RF	12.18	0.4587	0.3808
x, y, p	SVM	11.03	0.4164	0.3435
	RF	10.32	0.3849	0.3159
Δx,Δy	SVM	14.42	0.5467	0.4458
	RF	10.8	0.4	0.3266
Δx,$\Delta y, p$	SVM	12.74	0.5086	0.4187
	RF	11.09	0.3938	0.2994
x, y,Δx,Δy	SVM	13.44	0.5084	0.4418
	RF	11.83	0.4293	0.354
x, y,Δx,Δy,p	SVM	10.8	0.4393	0.3696
	RF	**10.03**	**0.36**	**0.2969**

System	Acc.	$\hat{C}_{llr}$	$\hat{C}_{llr}^{min}$
commercial	93.17	0.4134	0.2179
1st. non-commercial	84.81	0.5651	0.3511

SVM, is the best classifier, independently of the chosen features, in applications of handwriting recognition. For instance, the results in [22] show that SVM outperforms RF as a classifier, for the particular features (different from the ones chosen here) considered in that paper.

It can be also observed from Tables II and III, that the effectiveness of some time functions depends on the classifier being used. For Dutch data, whenever using pen coordinates with the SVM classifier the performance is strongly degraded, showing that the x and y coordinates are not robust (for this data) against the different classifiers. For Chinese data, while x and y coordinates appear to be more robust than Δx and Δy against the different classifiers, it is worth to highlight that, whenever using the pen pressure (independently of the classifier being used), the results improve, showing that pen pressure is robust against the different classifiers.

Analyzing the presented results, it can be inferred that the position information is likely to be better suited for the Chinese data than for the Dutch data. Chinese signature style is, in most of the cases, close to the Chinese handwriting style, consisting of one or more multi-trace ideograms, while Western signatures (Dutch signatures in this database) can adopt several different styles. Since Chinese characters usually convey their meaning through pictorial resemblance to a physical object, it is likely that the position information has more discriminative power than in the case of Dutch data.

The best result for the Dutch data, shown in boldfaced fonts in Table II, is obtained using the Δx,$\Delta y, p$ combination. Taking into account the above analysis, this makes sense since pen coordinates are not reliable features for this data, and including pen pressure information improves the results. The best result for the Chinese data, shown in boldfaced fonts in Table III, is obtained combining all the time functions available, viz, $x, y, \Delta x$,$\Delta y, p$. Chinese signatures appear to be more complex than Dutch ones, then it is not surprising that it is necessary to use more parameters to model the signature, in order to have better discrimination properties.

For the purposes of comparison, the results for the best commercial and non-commercial systems in the SigComp2011 competition are included in the last two rows of Tables II and III. It is worth to note that even tough the results are not as good as the corresponding to the best commercial system (*xyzmo*[7]), they would have ranked first among the non-commercial systems and second among all the participants. Finally, the results for the Dutch signatures are better than the ones for the Chinese signatures. This confirms the observations in [13] and indicates that Chinese data is more challenging and that lot of research has to be done on this type of data.

VI. CONCLUSION

Different combinations of the time functions associated to the signing process were studied in this paper for two different signature style datasets (Western signatures and Chinese signatures).

The experimental results showed that the contribution of the pen pressure when combined with the other time functions, improves the obtained performance for both the Dutch and Chinese data, independently of the classifier being used. In addition, in most of the cases, using the incremental pen coordinate variations Δx and Δy leads to a better performance than using directly the pen coordinates x and y.

Regarding the classification methods, the results obtained with RF are better than the ones obtained with SVM, independently of the signature style.

The experiments showed that the best combination of the time functions for the Dutch signatures was the one

[7]http://www.xyzmo.com

including Δx, Δy and p, while for the Chinese signatures was the one including x, y, Δx, Δy and p. It is not surprising the fact that more parameters are needed to model Chinese signatures since they appear to be more complex than the Dutch ones. The extra information (pen coordinates) needed can be explained by considering the fact that Chinese characters usually convey their meaning through pictorial resemblance to a physical object.

The use of the orthogonal polynomials to model the signatures proved to be a good choice, resulting in signature verification performances comparable to those of other state-of-the-art verification systems, tested on the same datasets. In addition, the proposed signature model would allow for a dimensionality reduction with respect to the case of using all the points in the time functions. Considering, for instance, a signature with about 1500 points and the optimal order of the Legendre polynomials which is 21, the dimensionality reduction would be in the order of $1500/22 \approx 68$.

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