# Strokes against Stroke
# Strokes for Strides

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13th International Conference

on Frontiers in Handwriting Recognition

ICFHR 2012, BARI SEPTEMBER 18-20

## SCRIBENS LABORATORY
## École Polytechnique

### Theory
Modelling of Trajectory Perception
and Human Movement Generation

### Applications
Automatic Processing of Handwriting

## PROJECTS

**BIOMETRY**
Automatic Signature Verification

**RECOGNITION**
On-line Handwriting Processing

**EDUCATION**
Handwriting Learning Tools

**BIOMEDICAL**
Neuromuscular Condition Evaluation

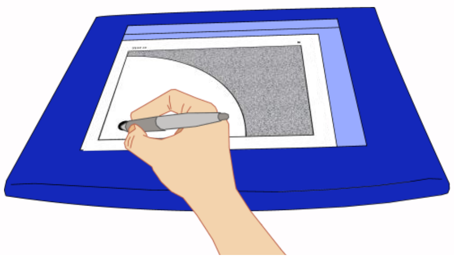
**DOCUMENTS**
Model-based Preprocessing

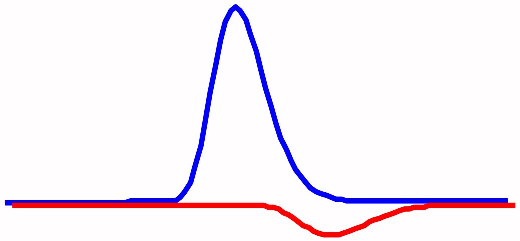
PART ONE

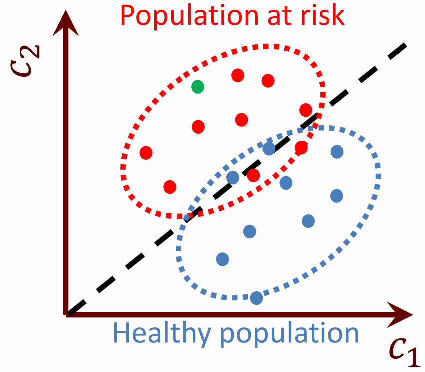
# STROKE AGAINST STROKES

CAN WE USE THE INFORMATION HIDDEN IN HANDWRITING STROKES TO PREVENT BRAIN STROKES?

# Vision — Long term goal









| $t_0$ | $D_1$ | $\mu_1$ | $\sigma_1$ | $D_2$ | $\mu_2$ | $\sigma_2$ |
|---|---|---|---|---|---|---|
| 0.18 | 150 | -1.85 | 0.37 | 14.5 | -1.42 | 0.13 |

➔ Patient highly at risk. Refer to a neurologist.

**Neuromotor test** → **Kinematic modeling** → **Diagnostic**

# MANY YEARS OF COLLABORATIVE WORK

- Prof. Bernard Clément, Statistician.
- Prof. Pierre A, Mathieu, Biomedical Engineering.
- Dr. Louise-Hélène Lebrun, Neurologist.
- Danielle Shashoua, Physiotherapist.
- Moussa Djioua, Ph.D.
- Christian O'Reilly, Ph.D.
- Anna Woch, Ph.D.
- Nicolas Leduc, M.Sc. A.
- Benoit Gervais-Laurendeau, Bac.El.Eng.
- Frédéric Nguyenphat-Therrien, Ing.jr.
- Alexandre Forget,Bac.El.Eng
- Jean-Philippe Morin, Ing.jr.
- David Filion, M.Sc.A.
- Daniel Lafrance, database exp.
- Claudiane Ouellet-Plamondon, Ing.jr., M.Sc.
- Luc Cloutier, Ing. M.Sc.A.

# TOPICS

1. Brain strokes
2. Handwriting strokes
3. Designing the experimental set up
4. Defining an experimental protocol
5. Collecting a database
6. Kinematic analysis
7. Statistical analysis
8. Main Results
9. Conclusion

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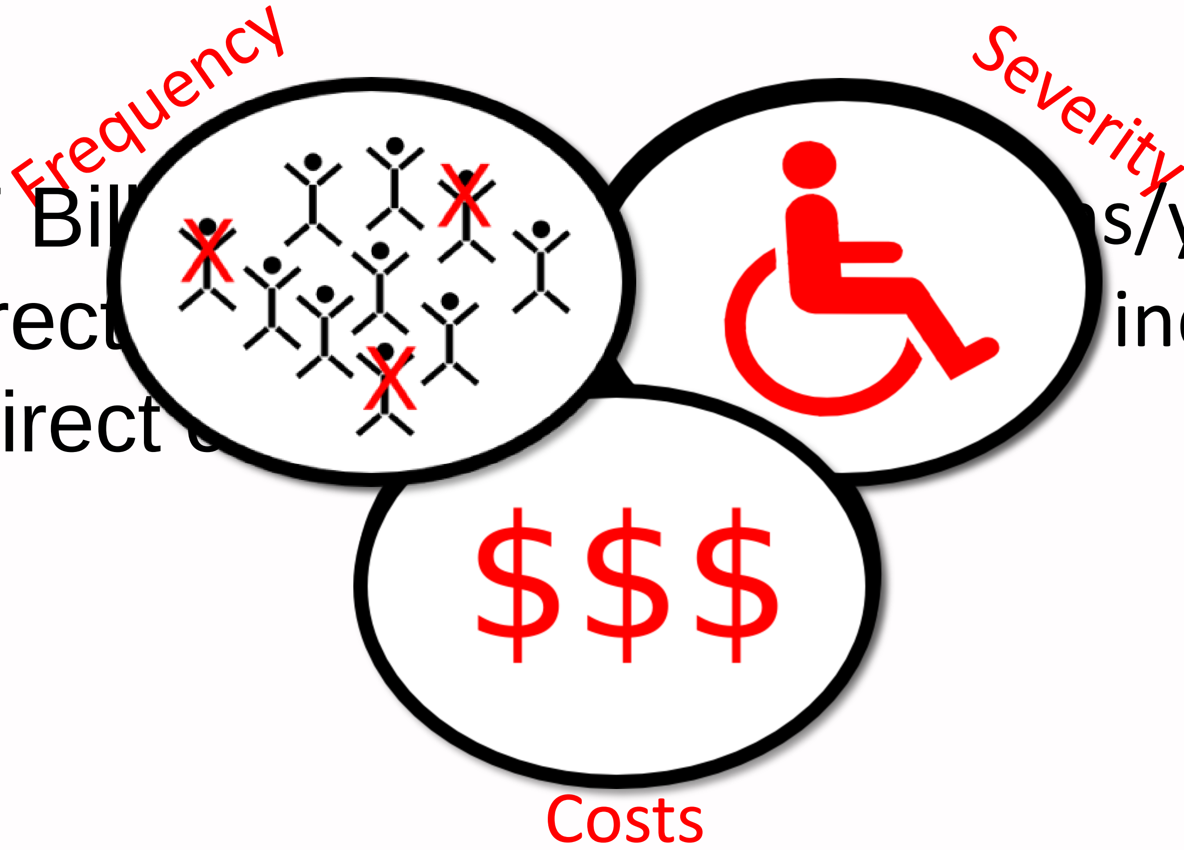
1 Vision

# Why focussing on Brain Strokes?

- 2.7 Bil… …s/year (direct … indirect) indirect c…




(Fondation des maladies du cœur, 2009; Lloyd-Jones et al., 2009; Kelly-Hayes et al., 2003)

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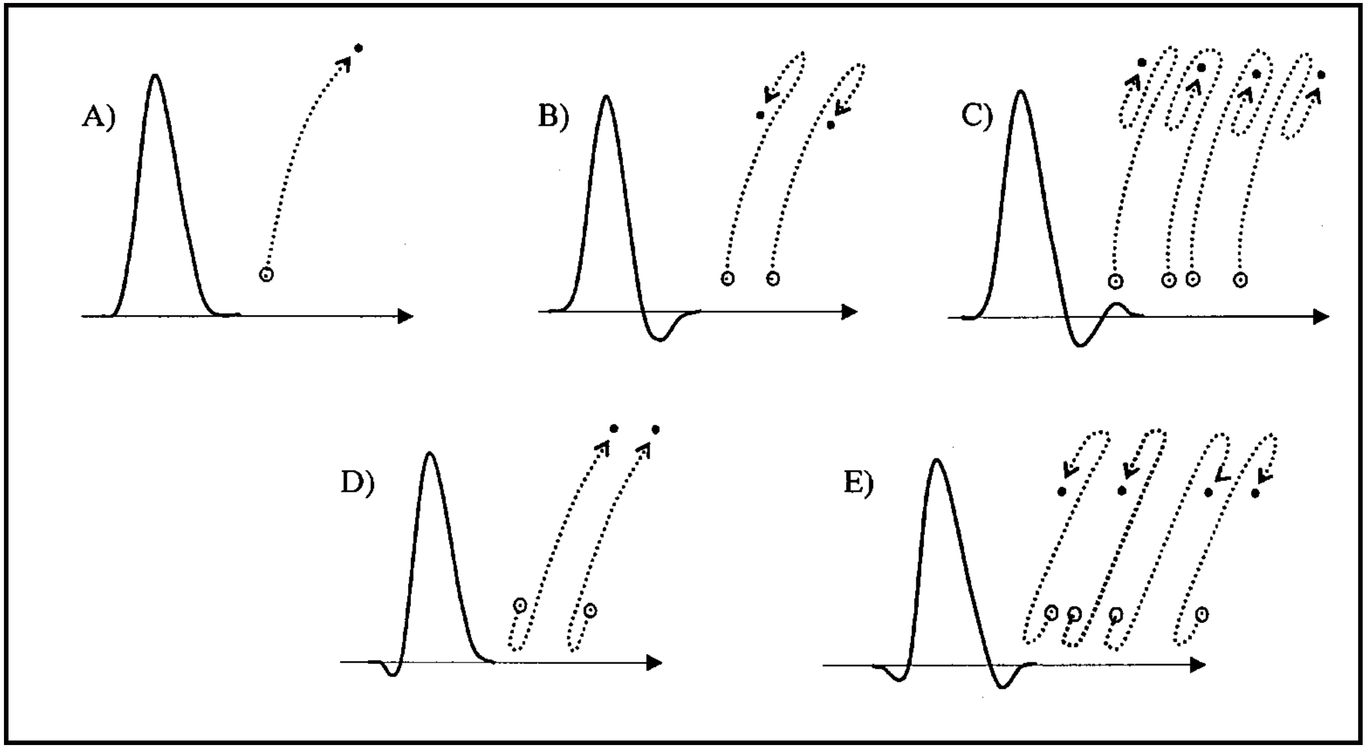
# What is a Handwriting Stroke?

**A fundamental unit of human movement**

**Our Fundamental Particle**

# Typical Velocity Profiles and Trajectories




WOCH, A., PLAMONDON, R., “Using the Framework of the Kinematic Theory for the Definition of a Movement Primitive”, Motor Control, vol 8, pp.547-557, 2004.

# Basic Characteristics of a Single Stroke

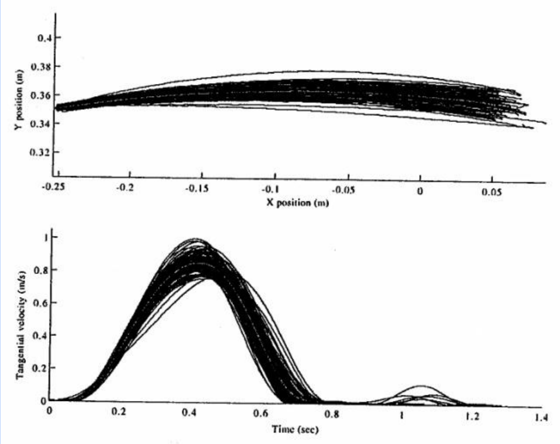



Figure : Typical trajectories and velocity profiles

- **No visual feedback**
- **Speed accuracy trade-offs**
- **Almost rectilinear trajectory of the end effector**
- **Asymetric bell-shaped velocity profile**
- **Up to two secondary velocity peaks**
- **Possible direction inversion at the begining and/or at the end**

Miyamoto H., Wolpert D.M., Kawato M., (2002) in: Biologically Inspired Robot Behavior Engineering, Springer-Verlag.

# Model Classification

- Criterion: Motor Control Basic Hypothesis
  - ❖ Equilibrium Point ( Feldman ,Bizzi, Hollerbach…)
  - ❖ Neural Networks (Bullock, Schomaker, Gangadhar…)
  - ❖ Optimization Principles ( Flash, Kawato, Alimi…)
  - ❖ Non-linear Dynamics ( Kelso, Athenes, Zazone…)
  - ❖ Proportionality and Convergence ( Plamondon)

# *Basic Hypothesis*

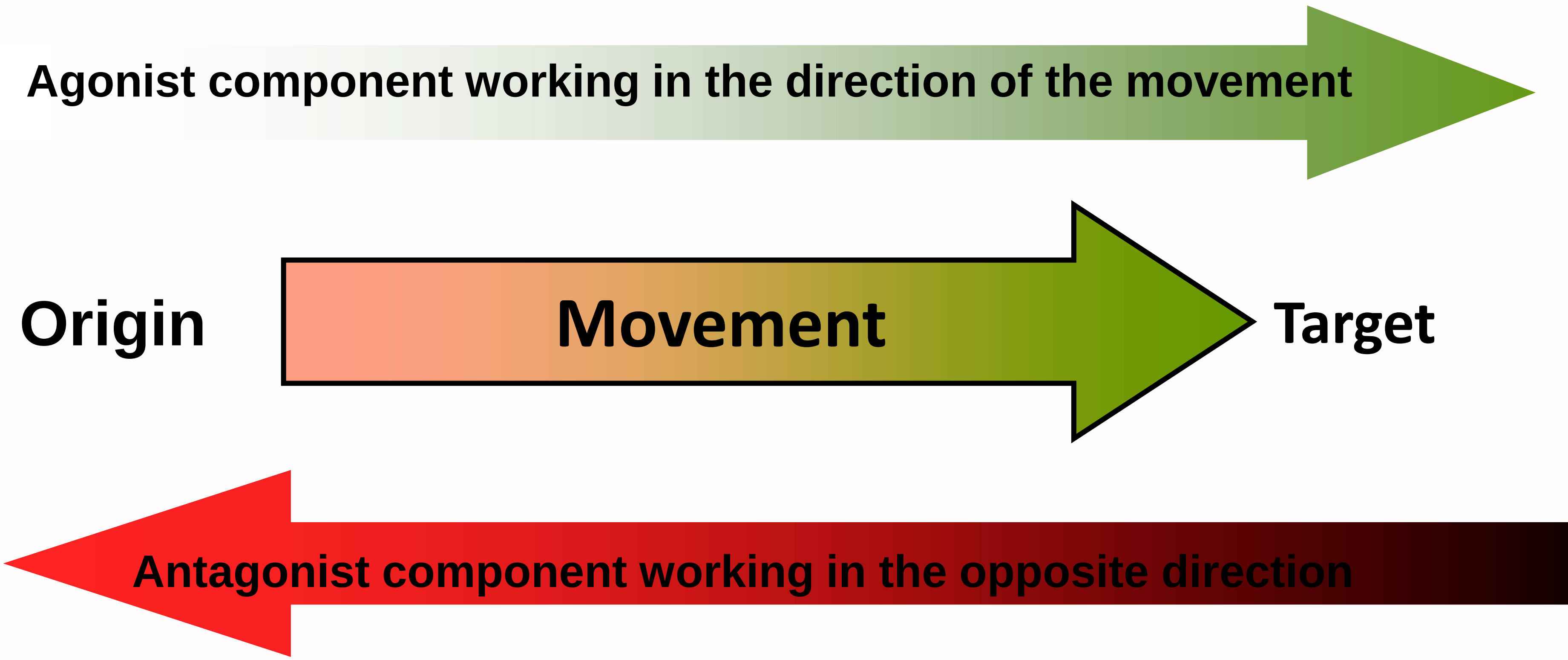
**The invariant properties of some characteristics of rapid human movements reflect the asymptotic behavior of complex systems, made up of a large number of coupled neuromuscular networks.**

→ **EMERGENCE FROM ASYMPTOTIC CONVERGENCE**

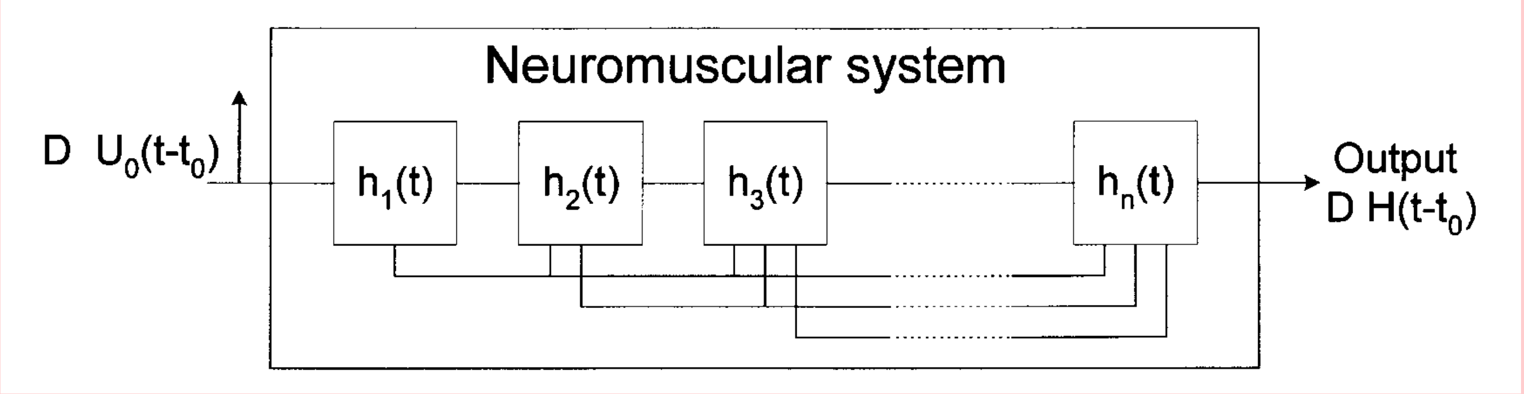
# *Basic Tool*

**The Central Limit Theorem can be used to point out emergent phenomena in these complex systems.**

# Agonist – Antagonist Synergy



- **Mathematical proof based on the Central Limit Theorem**
- **Convergence of the NMS impulse response towards a lognormal profile**



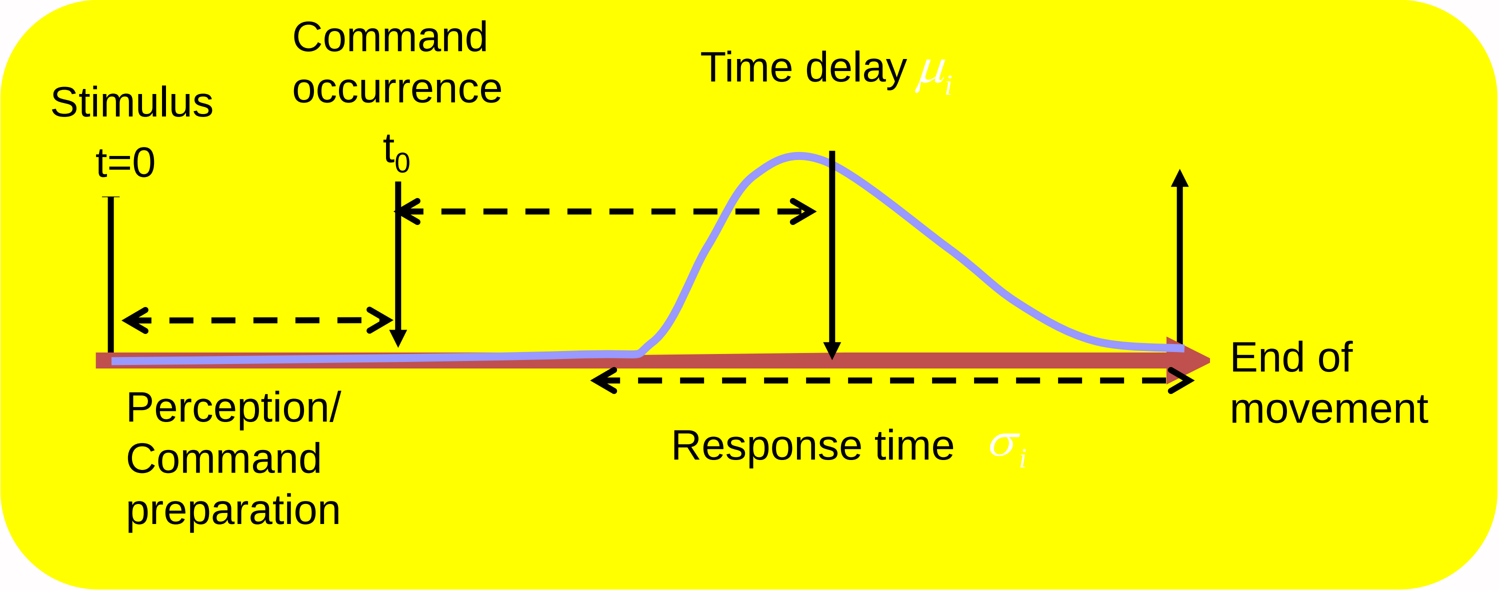

- **Hypothesis**

$$T_n = (1+\varepsilon_n)T_{n-1}$$

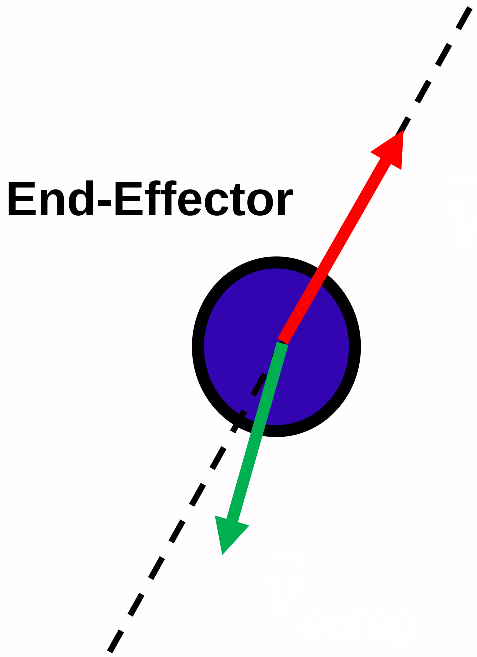
$$n \to \infty$$

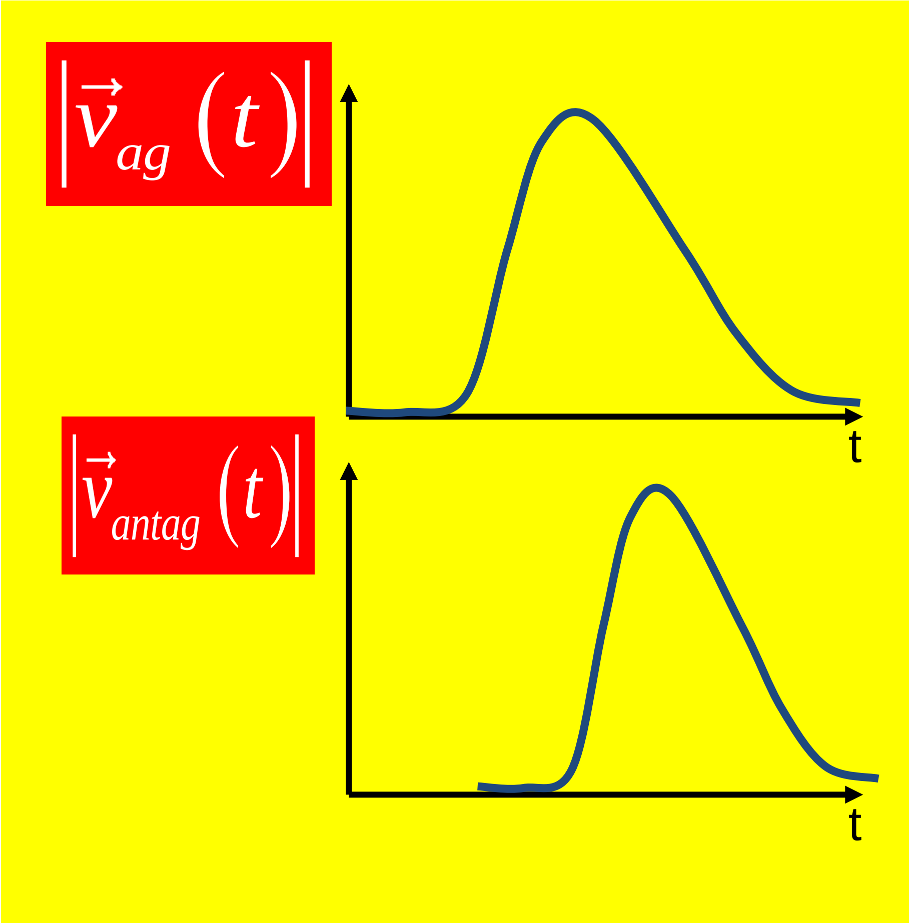
$$H(t-t_0) \Rightarrow \Lambda(t;t_0,\mu,\sigma^2)$$

# •*Temporal analysis of a movement component (agonist or antagonist)*



**Vectorial summation**



## Kinematic Theory : Delta-Lognormal model

**Velocity profile of a single stroke: Sigma-Lognormal Model**

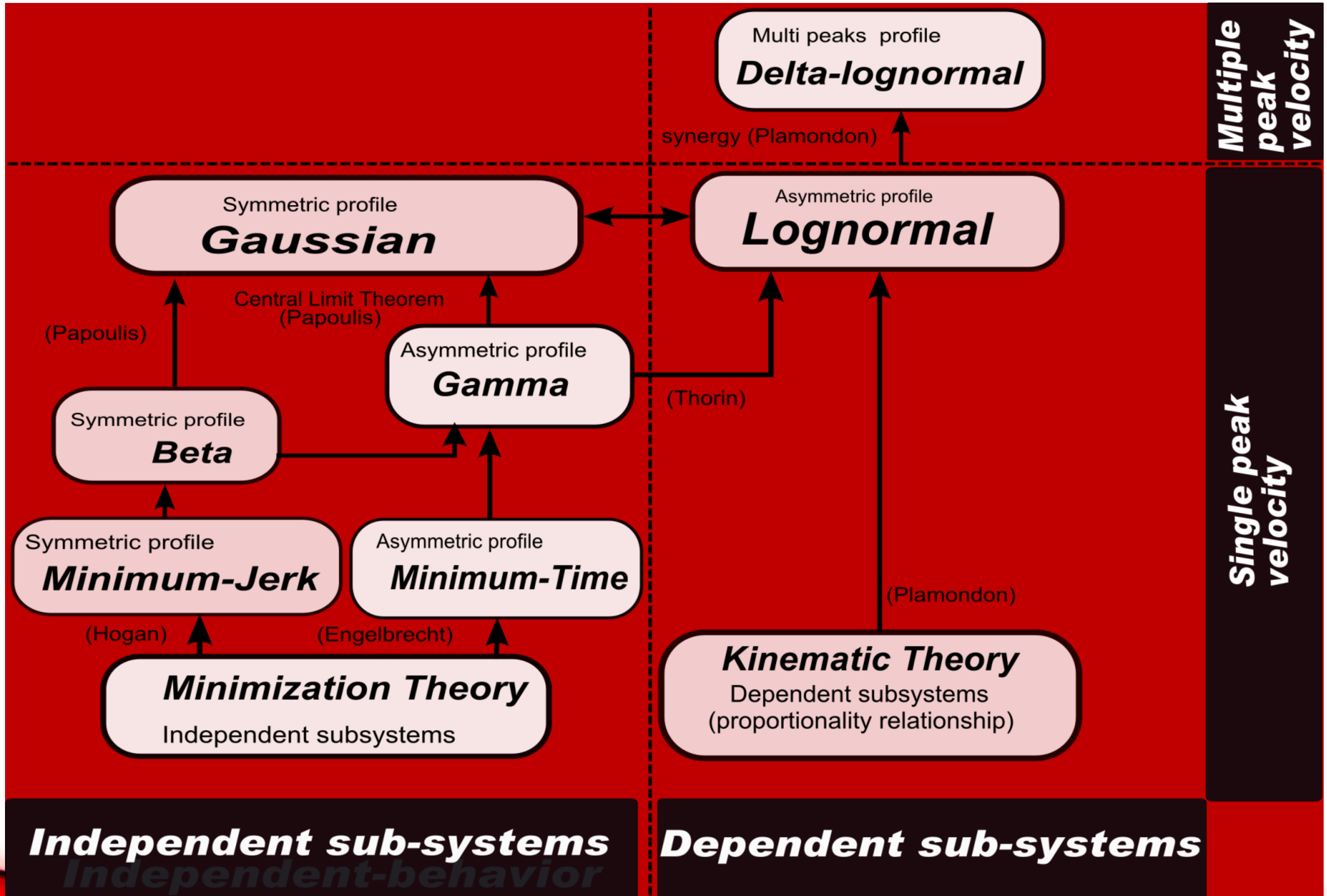
$$\vec{v}(t) = \vec{v}_{ag}(t) + \vec{v}_{antag}(t)$$

**Special case: perfect opposition of the agonist and the antagonist components**

$$v(t) = v_{ag}(t) - v_{antag}(t)$$

***Delta-Lognormal Model***

# MODEL COMPARISON




DJIOUA, M., PLAMONDON, R., “The Limit Profile of a Rapid Movement Velocity” , **Human Movement Science**, vol. 29, (2010), pp.48-61.

# A BRIEF PAUSE

**A STROKE IS THE IDEAL OUTPUT
OF A NEUROMUSCULAR SYSTEM
THE RESULT
OF AN EMERGING BEHAVIOR
PREDICTED
BY THE CENTRAL LIMIT THEOREM**

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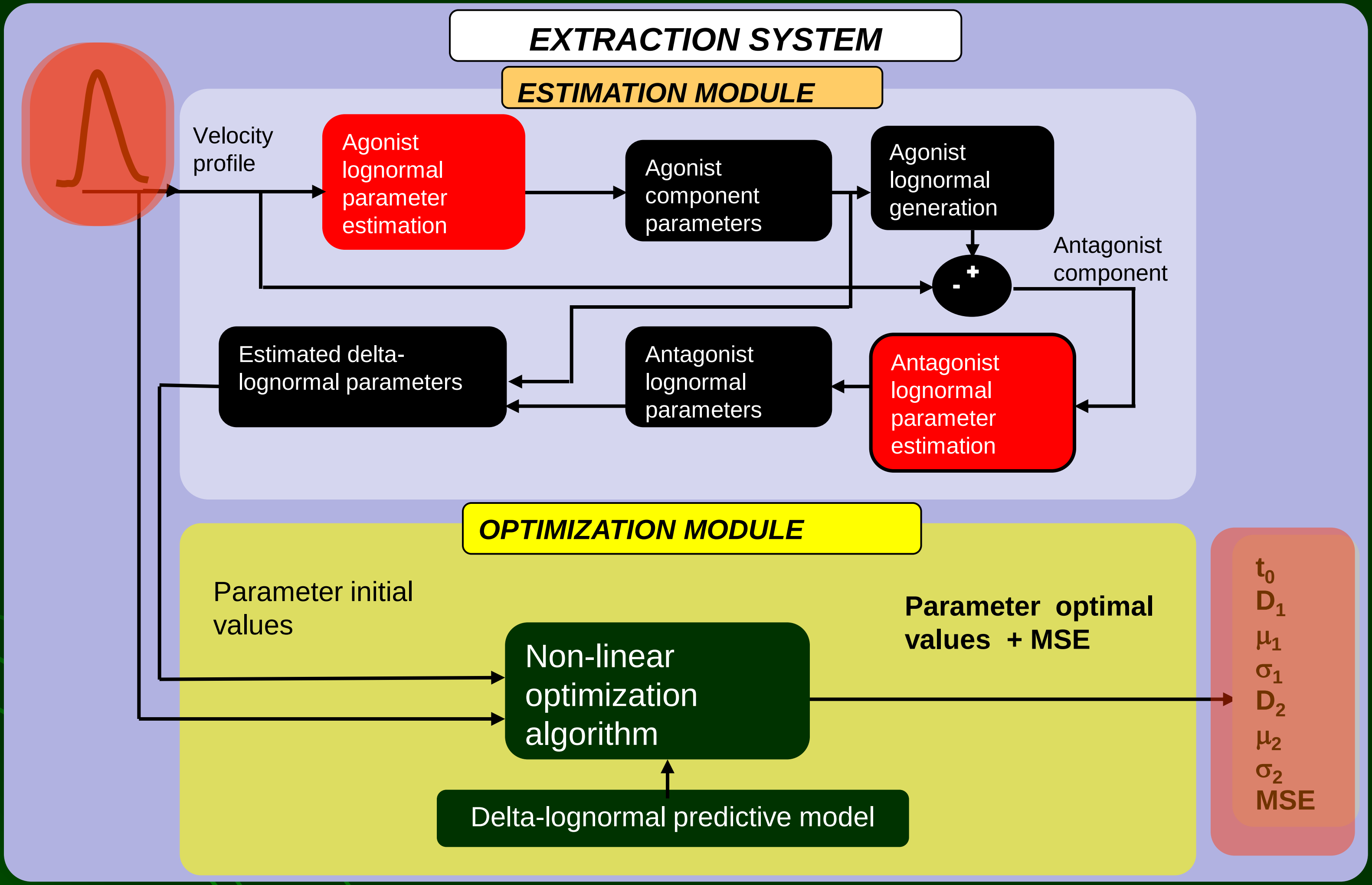
# SOFTWARE PROBLEMS

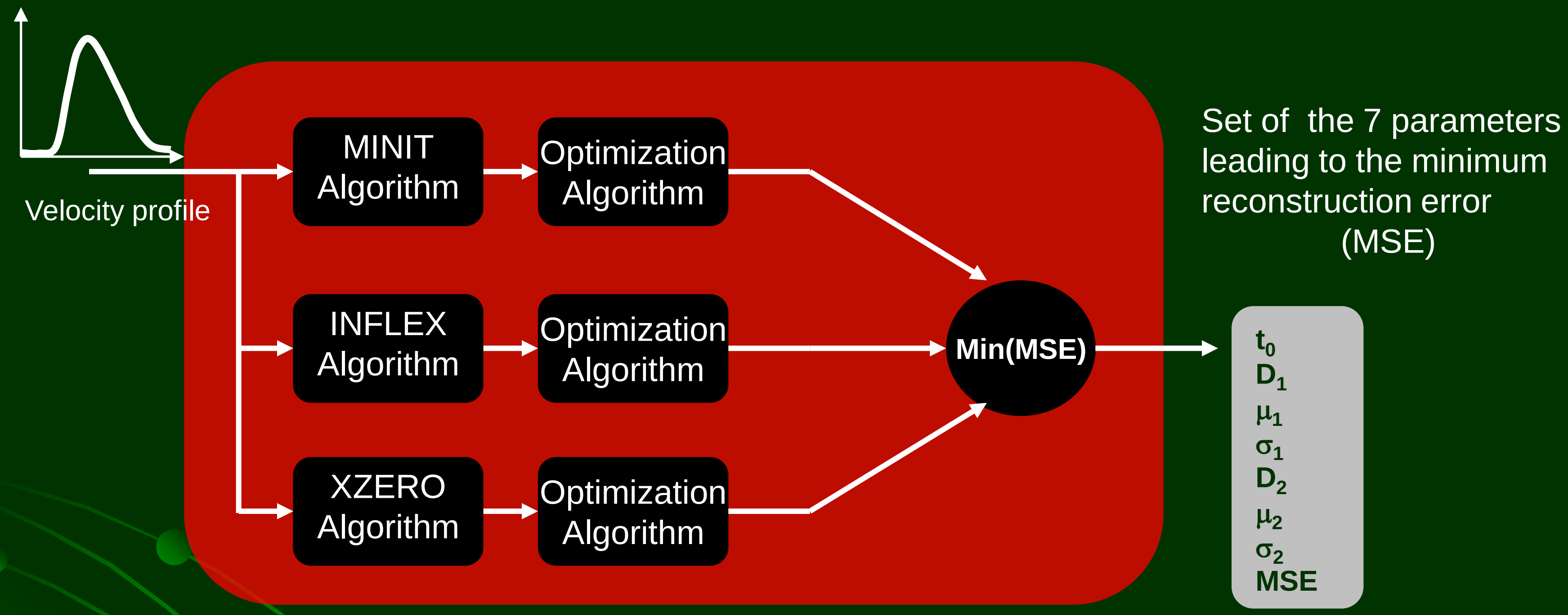
- Data acquisition:
  - Which signals?
  - Sampling frequency?
  - Data filtering?
  - Data base security?

- Parameter Extraction
  - Deterministic vs probabilistic approach
  - Step-wise approach (From Delta- to Sigma-)
  - Which algorithm?
  - How to validate local minimum solutions?
- User interface: ergonomy?

# Kinematic Theory : Extraction system : Architecture



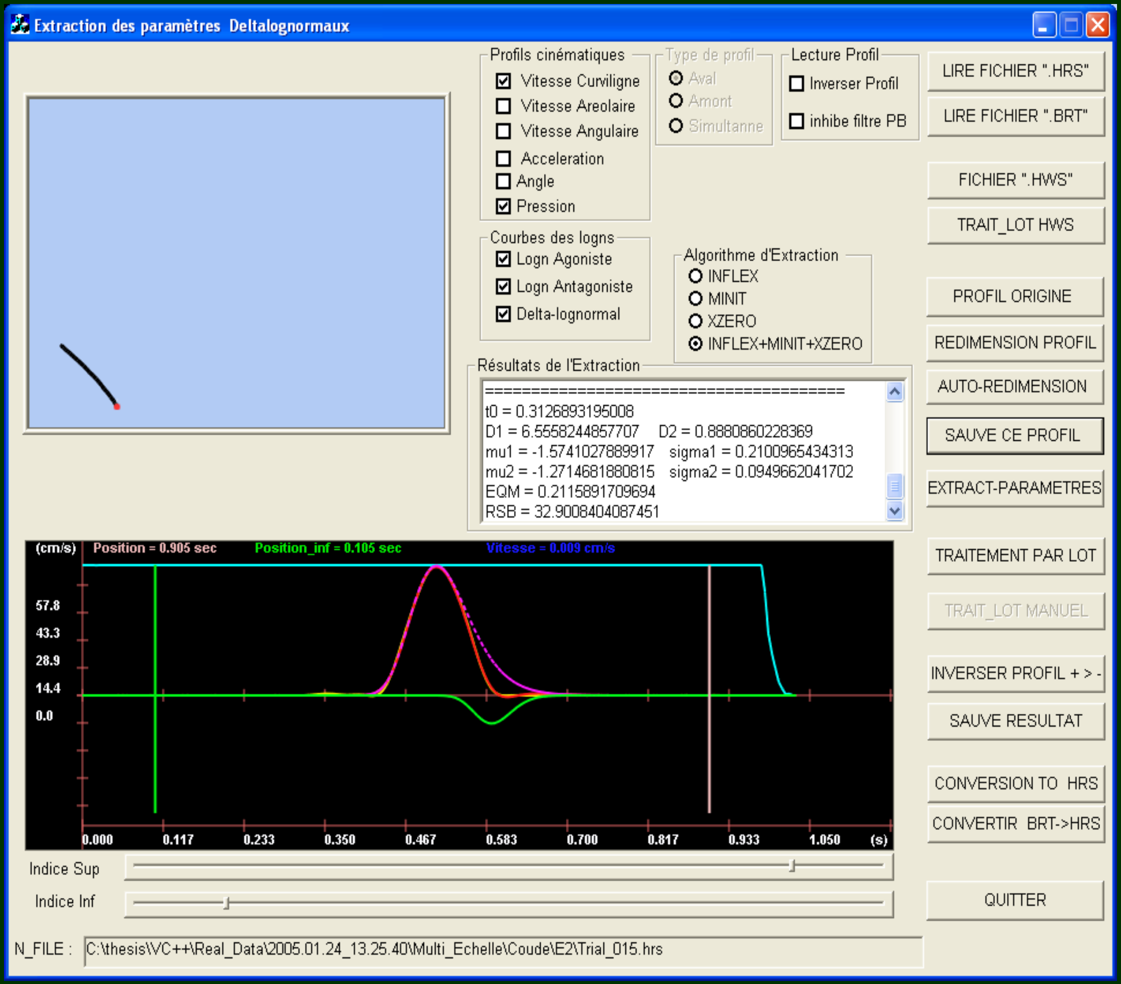
# Extraction system architecture



# Extraction System: final version

**Implementation of the three extraction algorithms**

+




**Extraction system characterization**

Djioua, M., Plamondon,R.
IEEE PAMI 2008 online

# Fitting — Four parameter extractors

## 1 INFLEX-INITRI-XZERO (IIX)

- ΔΛ representation (locally optimal)
- Fast reaching motion

M. Djioua and R. Plamondon, "A new algorithm and system for the characterization of handwriting strokes with delta-lognormal parameters," *IEEE Trans Pattern Anal Mach Intell*, vol. 31, pp. 2060-72, Nov 2009.

## 2 Branch and bound (B&B)

- ΔΛ representation (globally optimal)
- Fast reaching motion

C. O'Reilly and R. Plamondon, "A globally optimal estimator for the Delta-Lognormal modeling of fast reaching movements " *IEEE Trans. on System, Man and Cybernetics. Part B. Cybernetics*, in press.

## 3 Robust $X_0$

- ΣΔ representation
- Complex and arbitrary movements

C. O'Reilly and R. Plamondon, "Development of a Sigma-Lognormal representation for on-line signatures," *Pattern Recognition*, vol. 42, pp. 3324-3337, 2009.

## 4 Prototype based

- ΣΔ representation
- Complex and stereotypical movements
- Allow performing ANOVA of the ΣΔ parameters

O'Reilly and R. Plamondon, "Prototype-based methodology for the statistical analysis of local features in stereotypical handwriting tasks," presented at the International Conference on Pattern Recognition, Istanbul, Turkey, 2010.

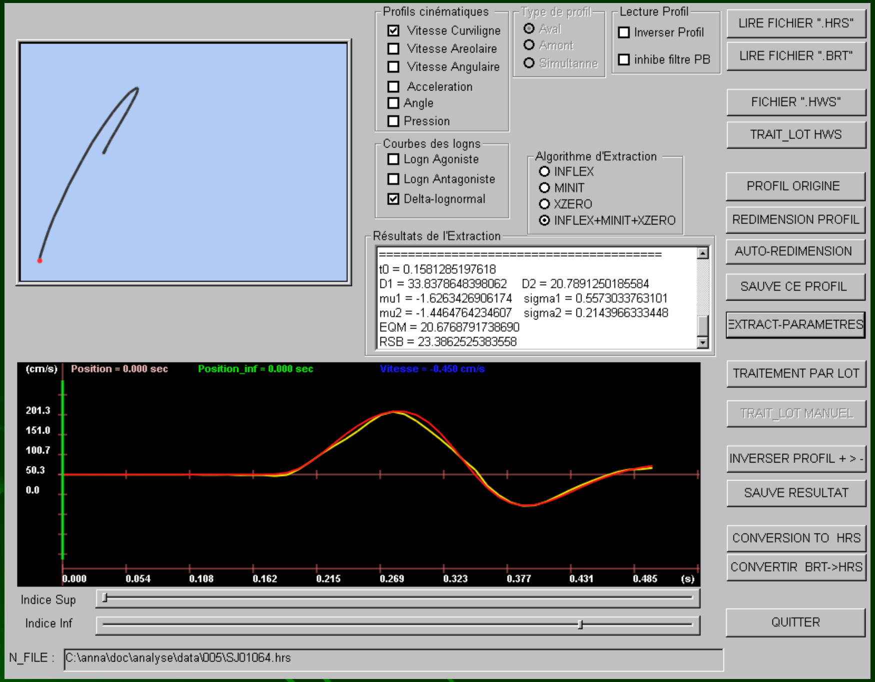
# HARDWARE PROBLEMS

- Digitizer vs instrumented pen?
- Portable computer vs pen pad?
- Design of a stimulus generator?
- Synchronisation and timing specifications
- Transportability and robustness
- Low costs
- Patient friendly

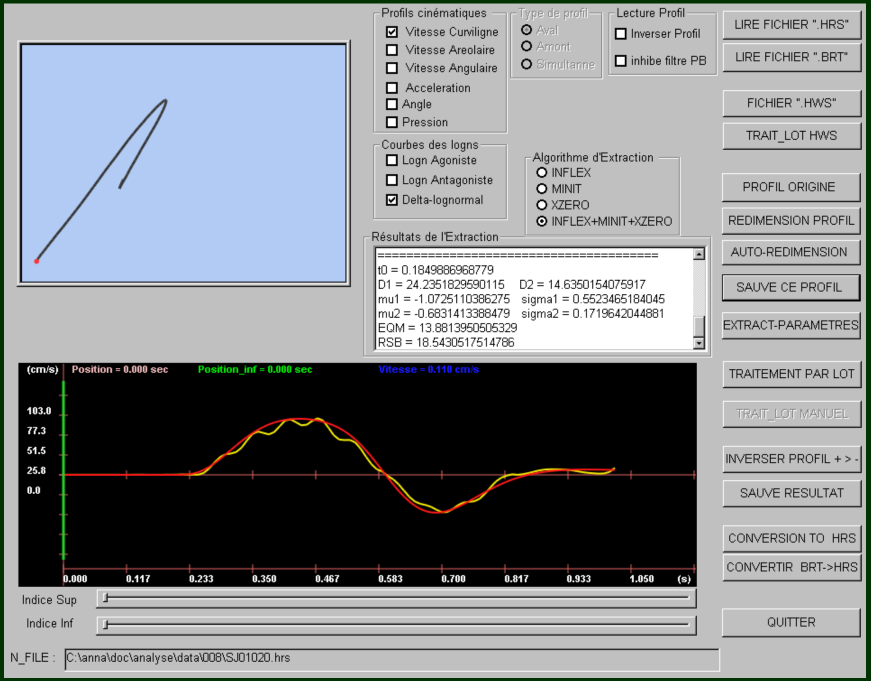
# Sign@medic System

# Typical Results

## Young subject




## Aged subject



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# Research Questions

- Context: a biomedical long term goal.
- Data anonymity and security?
- Pre-existing or new tasks?
- Which tasks? And why?
- How many tasks?
- How many repetitions?
- Success criteria and outlier rejection?

# Neuromotor testing

1. Registration:  Signatures
2. Auditory reaction time
3. Visual reaction time
4. Choice reaction time
5. Speed-accuracy trade-off
6. Triangular drawing
7. Maximal speed oscillations
8. Oscillations synchronized with an auditory metronome
9. Fatigue test: signature verification

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# Research Questions

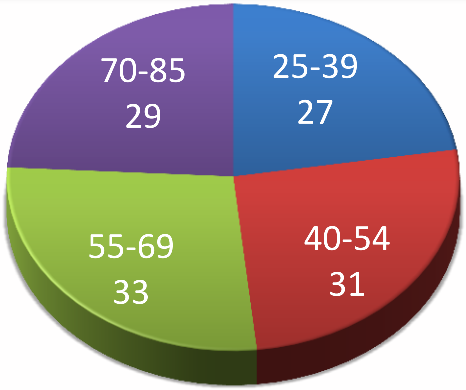
- Context: no familiarity with a computer environment for many participants.
- Understandability of the tasks?
- Preliminary practice?
- Duration of the experiment?
- Fatigue of the participants?
- Training of the experimenters?

# Standardizing the data acquisition

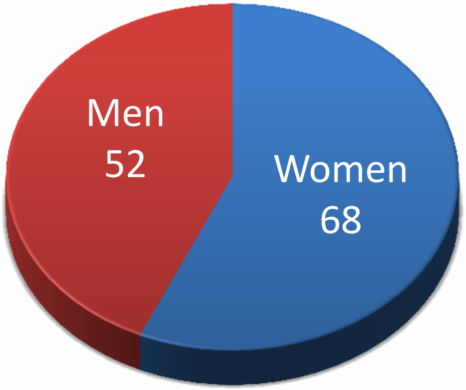
- Introductory leaflet for the subjects
- Experimenter guide
- Experimenter logbook
- Operator guide
- Operator logbook

# 5 Data — Sample (120 subjects)

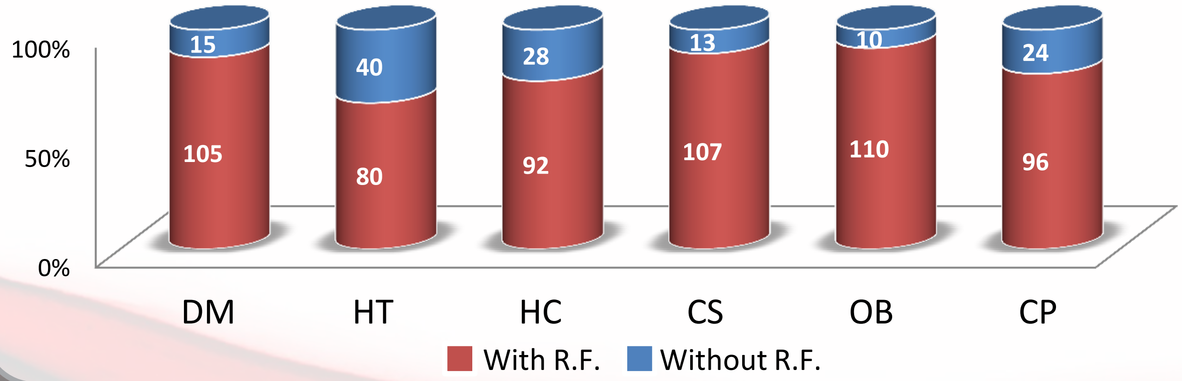
## Age




## Gender




## Brain stroke risk factors (R.F.)




DM: Diabetes mellitus
HT : Hypertension
HC : Hypercholesterolemia
CS : Cigarette smoking
OB : Obesity
CP : Cardiac problems

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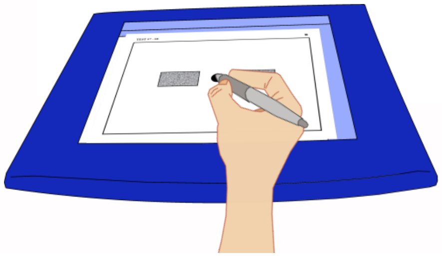
# Synergies: 3 types of movement

## 1 Reaching motion

Delta-lognormal (ΔΛ) modeling

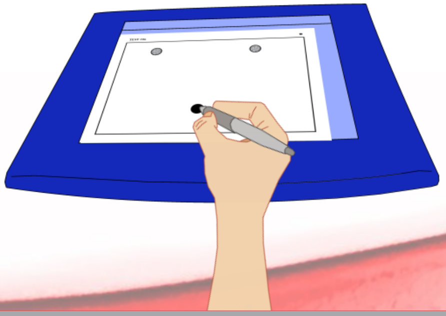
## 2 Oscillations

Omega-lognormal (ΩΛ) modeling

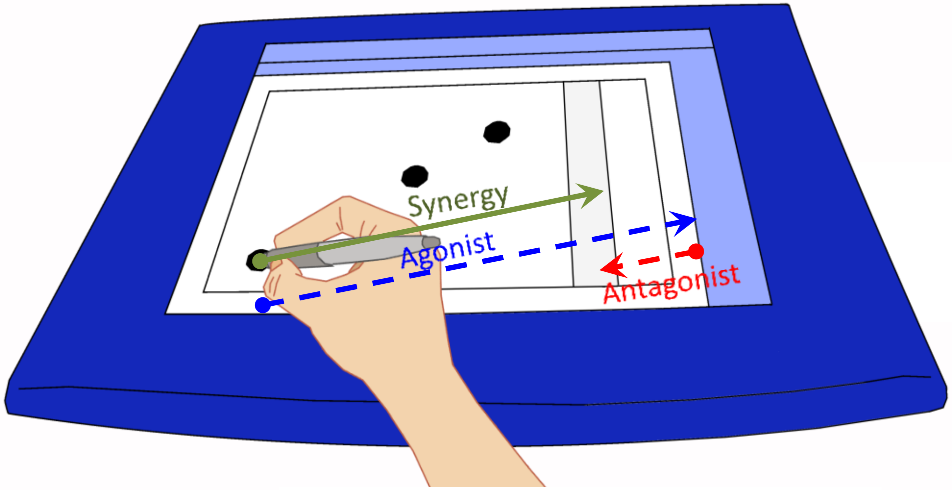


## 3 Complex motion

Sigma-lognormal (ΣΛ) modeling



# Delta-Lognormal



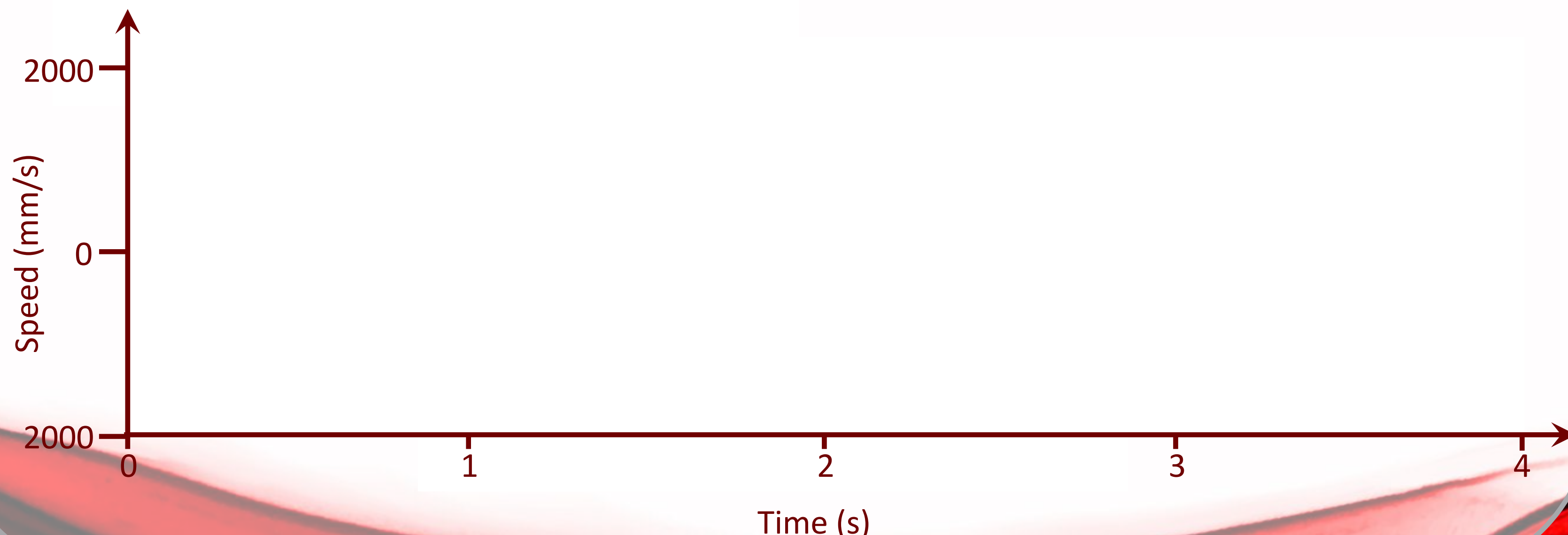


$$
\begin{aligned}
v_t(t) &\approx D_1\Lambda(t - t_0;\ \mu_1, \sigma_1) \\
&\quad - D_2\Lambda(t - t_0;\ \mu_2, \sigma_2) \\
&= \Delta\Lambda(t)
\end{aligned}
$$

# Modeling — Omega-Lognormal

$$v_t(t) \approx \sum_{\substack{i=1 \\ v_{t\,max\,i}>0}}^{\xi} D_{1i}\,\Lambda(t - t_{01i}; \mu_{1i}, \sigma_{1i}) - \sum_{\substack{i=1 \\ v_{t\,max\,i}<0}}^{\xi} D_{2i}\,\Lambda(t - t_{02i}; \mu_{2i}, \sigma_{2i})$$

$$= \Omega\Lambda$$



## 2D representation

A complex motion trajectory can be described by the motion speed ($v_t(t)$) and direction ($\varphi(t)$).

## Component representation and synergy

- A neuromotor component is acting around a pivot point.
- Direction of the i$^{th}$ component trajectory :

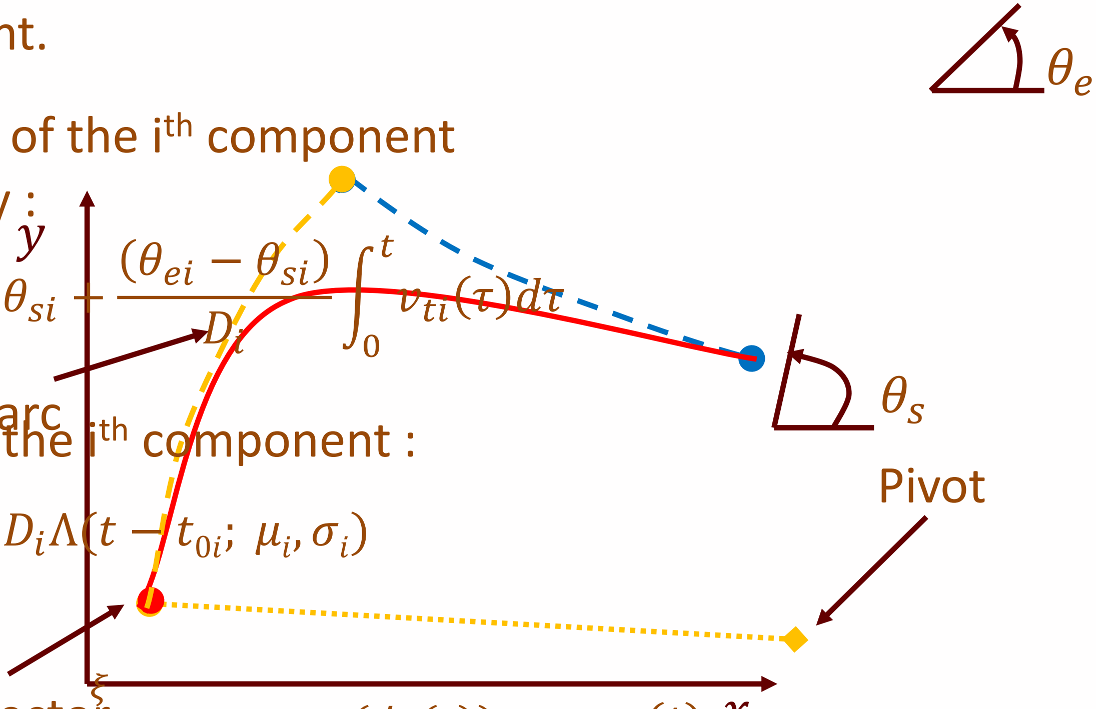
$$\varphi_i(t) = \theta_{si} + \frac{(\theta_{ei} - \theta_{si})}{D_i} \int_0^t v_{ti}(\tau)d\tau$$

- Speed of the i$^{th}$ component :

$$v_{ti}(t) = D_i \Lambda(t - t_{0i};\ \mu_i, \sigma_i)$$

- Synergy :

$$v(t) = \sum_{i=1}^{\xi} v_{ti}(t) \begin{bmatrix} \cos(\phi_i(t)) \\ \sin(\phi_i(t)) \end{bmatrix} = \begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix}$$



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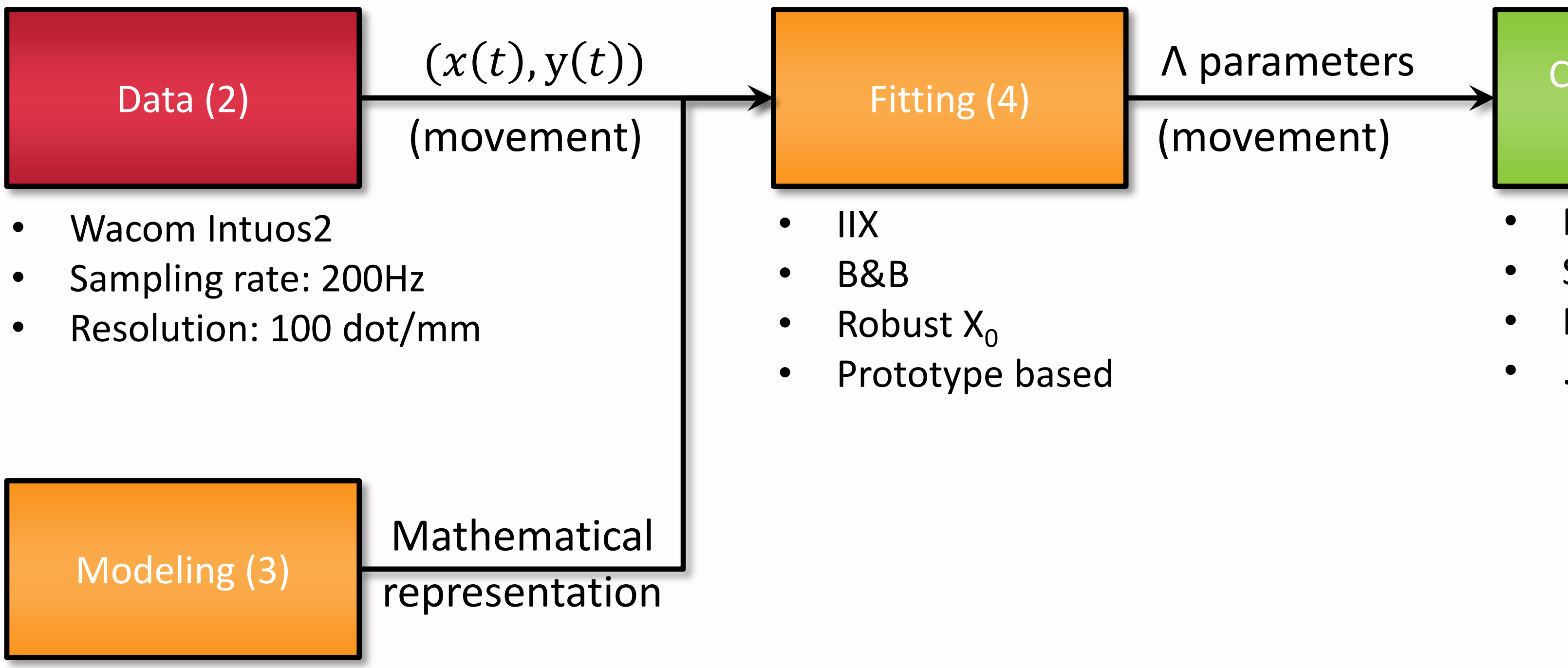
# Diagnostic




Data (2):

- Wacom Intuos2
- Sampling rate: 200Hz
- Resolution: 100 dot/mm

Fitting (4):

- IIX
- B&B
- Robust $X_0$
- Prototype based

Modeling (3):

- Delta-Lognormal
- Omega-Lognormal
- Sigma-Lognormal

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# Main Results

## Classification/diagnostic results (AUC)

| | Reaction time | | | | Speed-accuracy trade-off | Triangular drawing | Maximum speed oscillations | signatures | $\bar{x}$ |
|---|---|---|---|---|---|---|---|---|---|
| | Visual | Auditory | Choice | Combined | | | | | |
| **Diabetes** | 0.85 | 0.82 | 0.89 | 0.88 | 0.85 | 0.82 | 0.76 | 0.82 | 0.84 |
| **Hypertension** | 0.76 | 0.76 | 0.76 | 0.81 | 0.74 | 0.80 | 0.77 | 0.76 | 0.77 |
| **Hypercholesterolemia** | 0.81 | 0.78 | 0.73 | 0.83 | 0.75 | 0.73 | 0.66 | 0.69 | 0.75 |
| **Cigarette smoking** | 0.69 | 0.82 | 0.72 | 0.78 | 0.71 | 0.70 | 0.60 | 0.34 | 0.67 |
| **Cardiac problems** | 0.81 | 0.82 | 0.85 | 0.85 | 0.80 | 0.81 | 0.74 | 0.82 | 0.81 |
| **Obesity** | 0.78 | 0.88 | 0.85 | 0.85 | 0.73 | 0.68 | 0.75 | 0.73 | 0.78 |
| $\bar{x}$ | 0.78 | 0.81 | 0.80 | 0.83 | 0.76 | 0.76 | 0.71 | 0.69 | 0.77 |

# SUMMARY

- As area under the ROC curve of 0.60 to 0.89 have been obtained on the prediction of six of the principal CVA risk factors using only information extracted from the movements, there is a definitive relationship between the presence of stroke risk factor and the characteristics of human movements. Although, a large part of it may be attributed to the effect of the age and the gender, there are convincing evidences that these two factors does not account for all of it. Therefore, human movements seem to contain supplementary information related to the susceptibility of eventually suffering from a Brain Stroke which is neither attributable to the age nor the gender.

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# 8 Conclusion

## Outcomes

1. Design of a complete framework for neuromuscular disorder diagnostic from movement analysis

2. Study of the links between brain stroke susceptibility and human movements (Handwriting stroke)

STROKES AGAINST STROKES

3. Design of new tools  for human movement analysis which can be also applied to other problems

# On-Going Collaborations

| | University | Research Project |
|---|---|---|
| 1 | Université de Montréal (Julien Doyon) | Effects of Cardiorespiratory Physical Exercise Motor Skill Learning in Parkinson's disease. |
| 2 | Louisiana State University (Arend van Gemmert) | Evaluation of the fine motricity of Parkinsonian Patients. |
| 3 | Université des Antilles et de la Guyane (Céline Rémi) | Neuromotor problem screening to help Children to learn Handwriting. |
| 4 | Université des Antilles et de la Guyane (Lydia Foucan) | Type I and Type II Diabete Discrimination. |

# 6 Conclusion

## Future work

Short term

1. Classifier combination
2. Latent variable modeling
3. Corroboration on new transversal data
4. Prospective study

Long term

# The Practical Lessons Learned (So Far…)

- Reduce the number of tasks
- Increase the number of repetitions
- Design a new generation of the system using Tablet-PC ?
- Reject outliers on the spot
- Increase the number of participants
- Improve the health questionnaire
- Longitudinal and international studies

# Theoretical Lesson Learned

- Pattern recognition is an efficient paradigm.
  - Start from scratch:
    representation, mapping and interpretation
  - Freedom of thought
  - Occam razor: Does it works?
- Emergent solutions from powerful models.
- Good ideas can be applied to other problems.
- Can we use patterns recognition methods in our quest for a better understanding of Nature.

PART TWO

# STROKE FOR STRIDES

# What is a Stride?

A long, decisive step

(Oxford DictionaryThesaurus)

PART TWO

# STROKE FOR STRIDES

CAN WE USE THE SAME PATTERN RECOGNITION APPROACH TO SOLVE OTHER TYPES OF PROBLEMS?

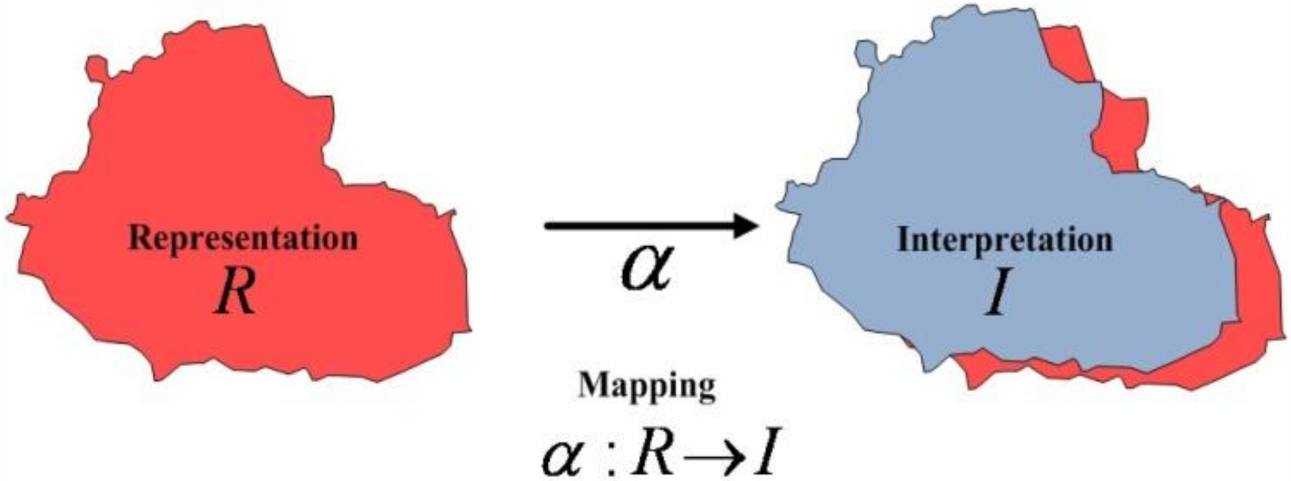
## Vision | Long term goal

**APPLYING**
**PATTERN RECOGNITION**
**TECHNIQUES**
**TO TRY BRIDGING THE GAP**
**BETWEEN**
**GENERAL RELATIVITY**
**AND**
**QUANTUM MECHANICS**
**PROVIDING SOME NEW INSIGHTS**
**THE UNIFICATION OF PHYSICS**

# TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
3. Incorporating quantum mechanics
4. A symmetric geometry
5. An axisymmetric geometry
6. Three supplementary emerging interactions
7. From stars to galaxies… to the Universe
8. Take home messages

# Statistical Pattern Recognition




Patterns are generated by a probabilistic system

# Statistical Pattern Recognition

- REPRESENTATION

  scaled features ⇔ N-dimensional space
  object⇔ random vector

- INTERPRETATION

  class ⇔ a cluster defined by a density function

- MAPPING

  class delimitation ⇔ discriminating function

# TOPICS

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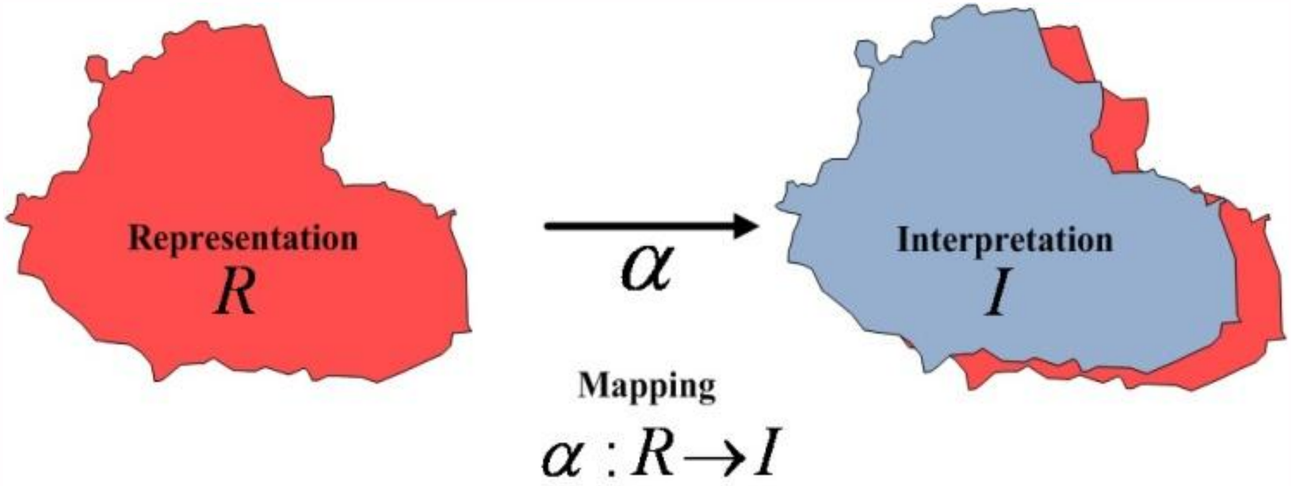
# Einstein's equation: G=KT




Two information spaces must be analyzed and compared.

# First information space: the structure of the manifold

- REPRESENTATION
  - coordinates +metrics ⇔ 4-dimensional space
  - metric quantified coordinate ⇔ arbitrary specific feature of the manifold
- MAPPING
  - Einstein tensor: G
- INTERPRETATION
  - 16 component curvature space

# Second information space: the content of a manifold

- REPRESENTATION
  - coordinates +metrics ⇔ 4-dimensional space
  - metric quantified coordinates ⇔ localization of events
- MAPPING
  - Momentum-Energy tensor: T
- INTERPRETATION
  - Mass-energy density, energy flux, momentum density and stress components

# Einstein’s equation

$$G = KT$$

- The Einstein’s equation can be seen as making a link between the two interpretation spaces.
- BUT, according to the statistical pattern recognition paradigm, these interpretation spaces could be given a probabilistic meaning…How?

# TOPICS

1. A statistical pattern recognition approach
2. Putting general relativity into a probabilistic context (The interdependence principle)
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# Interdependence principle

*Spacetime curvature (S) and matter-energy density (E) are two inextricable information spaces defining the physically observable universe (U); they must be mutually exploited to describe any subset* $U_i$ *of this universe. In terms of expectations, the probability of observing a subset* $(U_i)$ *is:*

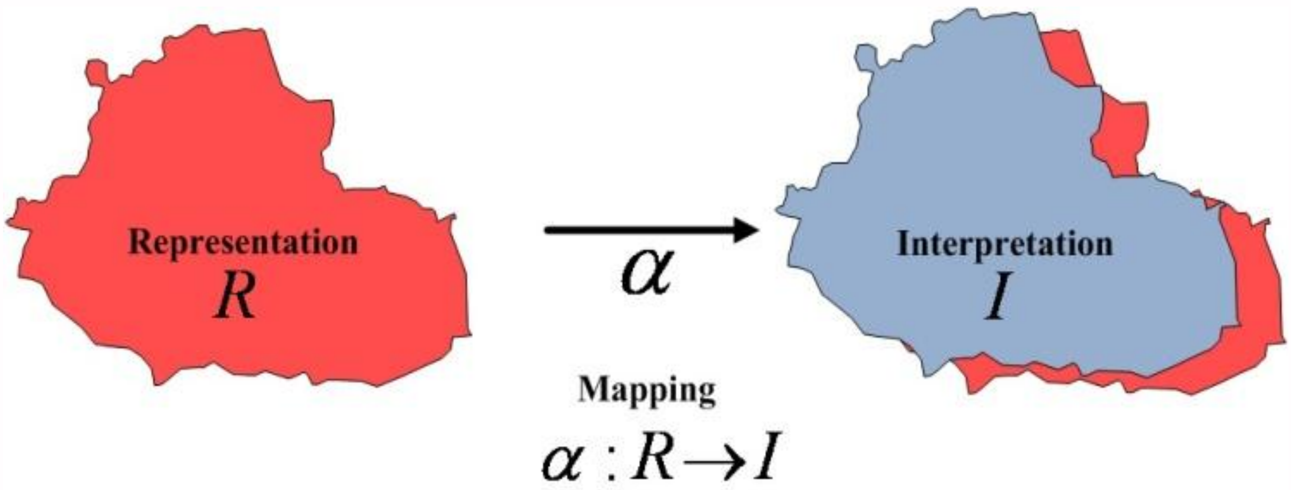
$$P(U_i) = P(S_i, E_i) < 1$$

# Corrolary

The probability of observing and describing a given subset of the universe P($U_i$), that is the joint probability of P($S_i$,$E_i$), can be studied from two equivalent *modi operandi*: either by analyzing the structure of the spacetime as an interpretation space associated with an *a priori* given matter-energy density or by analyzing the matter-energy density as an interpretation space associated with an *a priori* given spacetime structure.

# In terms of Bayes'law…

(conditionnal probabilities)




$$\mathrm{P}(U_i) = \mathrm{P}(S_i, E_i) = \mathrm{P}(S_i/E_i)\mathrm{P}(E_i) = \mathrm{P}(\mathrm{E}_i/\mathrm{S}_i)\mathrm{P}(S_i)$$

$$f(S_i/E_i)f(E_i) = f(\mathrm{E}_i/\mathrm{S}_i)f(S_i)$$

$$f(S_i/E_i) = f(\mathrm{E}_i/\mathrm{S}_i)\frac{f(S_i)}{f(E_i)}$$

# A link with Einstein's law?

$$f(S_i/E_i) = f(\mathrm{E}_i/\mathrm{S}_i)\frac{f(S_i)}{f(E_i)} \Leftrightarrow G = KT$$

$$f(S_i/E_i) = k_1 trG$$

$$\frac{f(S_i)}{f(E_i)} = k_2 trT$$

$$f(\mathrm{E}_i/\mathrm{S}_i) = ?$$

# A Potential Pathway…

$f(E_i \ / \ S_i)?$

Reflects the probability of presence
of
a given energy momentum density

In Quantum Mechanics, the wave function $\psi_{E_i}$
can be used to compute
the probability of presence of a given particle

$$\psi^*_{E_i}\psi_{E_i} = f_\psi(S_i)$$

# Introducing Quantum Mechanics into General Relativity

$$f(\mathrm{E}_i/\mathrm{S}_i) = k_3 \psi^*_{E_i} \psi_{E_i} = k_3 f_\psi(S_i)$$

$$G = \frac{k_2 k_3}{k_1} f_\psi(S_i) T$$

$$f_\psi(S_i)?$$

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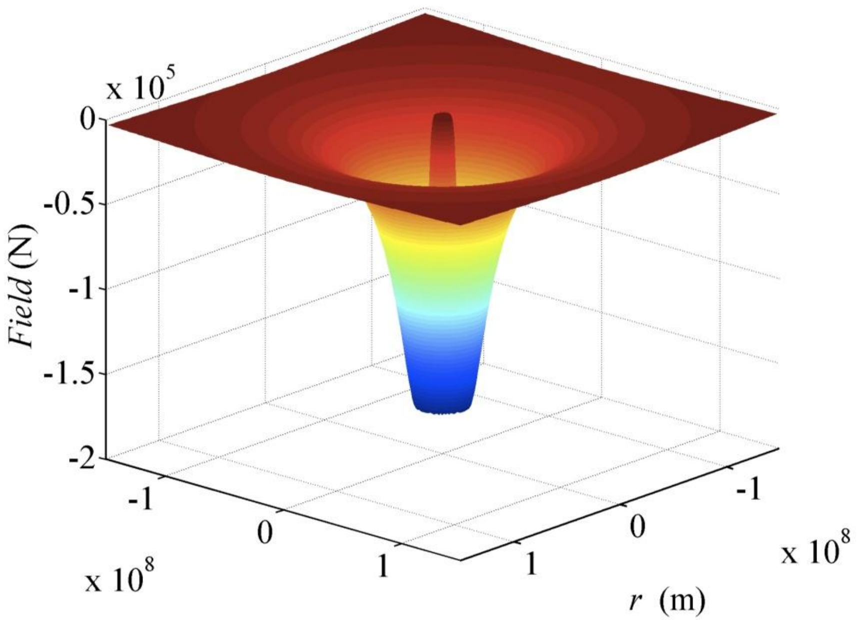
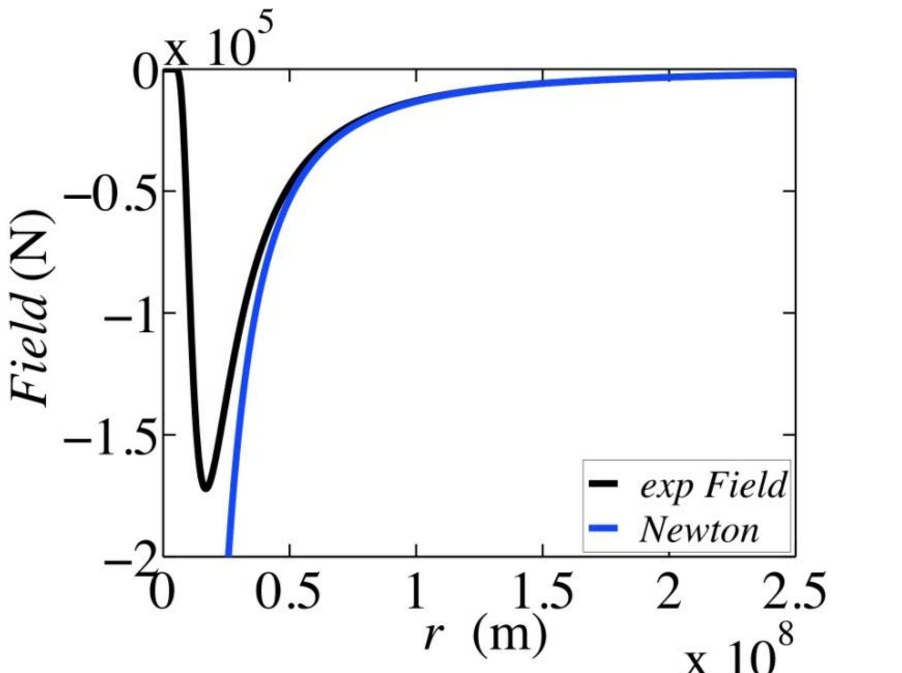
# Estimating the probability of presence

- Building a star from scratch by adding numerous identical particles ($N \rightarrow \infty$), each one with its own wave function, density function and associated space-time, as seen from a locally flat tangent space.
- Making the convolution of their corresponding density functions.
- The central limit theorem predicts that the ideal form of the global probability density $f(\boldsymbol{x})$ will be a Gaussian multivariate function.

# Emergence of Newton's law of gravitation: the field

$$g(r) = -\left|\vec{\nabla}\Phi\ r\right| = -\frac{2KMc^4}{4\pi\sigma^3 r^2}\exp\left(-\frac{\sigma^2}{2r^2}\right) \qquad g(r) \cong -\frac{2KMc^4}{(4\pi\sigma)^3 r^2} = -\frac{GM}{r^2}$$

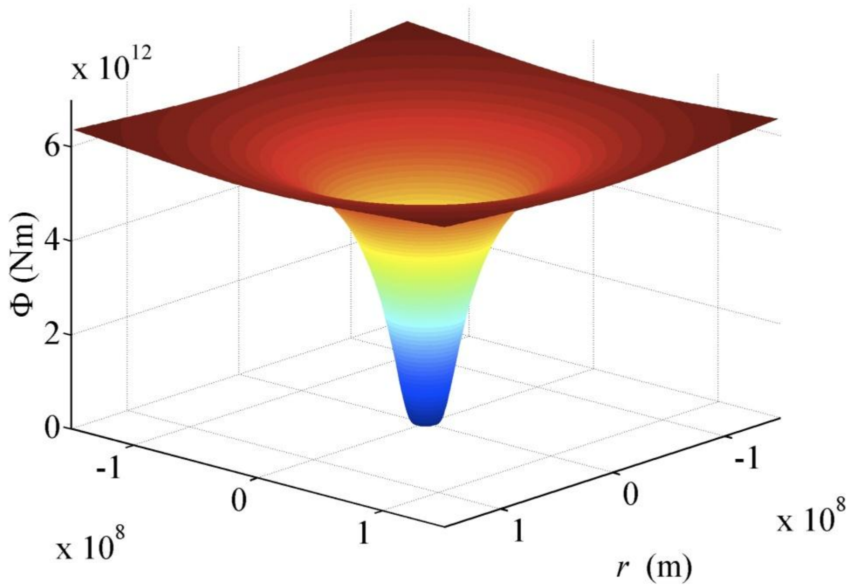


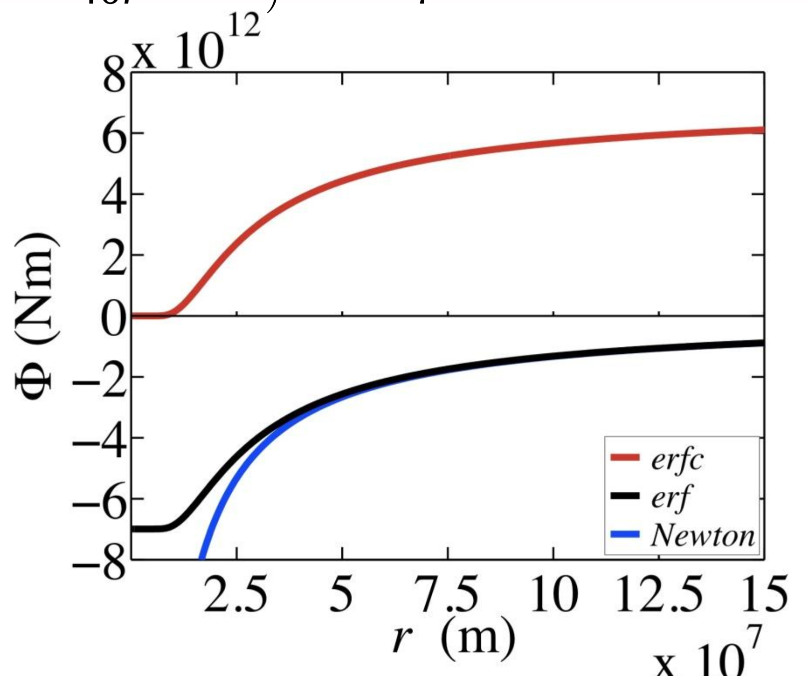



# Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{4\pi\sigma^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) = \Phi_{erfc}(r)$$

$$\Phi_{erf}(r) = -\frac{2KMc^4}{4\pi\sigma^3}\left(\frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \ldots\right) \cong -\frac{GM}{r}$$



# Brief pause

- According to the present pattern recognition paradigm, the Newton's law is not empirical. It is an approximation of a more general law. It can be seen as an emerging phenomenon.
- The resulting *erfc* potential can  be incorporated in a metric to study statically symmetric system.

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# The symmetric metric and the field equation

$$ds^2 = \left[1 + \frac{2}{c^2} GM\left(\frac{\sqrt{\pi}}{\sigma\sqrt{2}}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c^2 dt^2$$
$$- \left[1 + \frac{2}{c^2} GM\left(\frac{\sqrt{\pi}}{\sigma\sqrt{2}}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

- No coordinate singularity
- No intrinsic singularity
- Temporal offset at infinity
- Radial delays

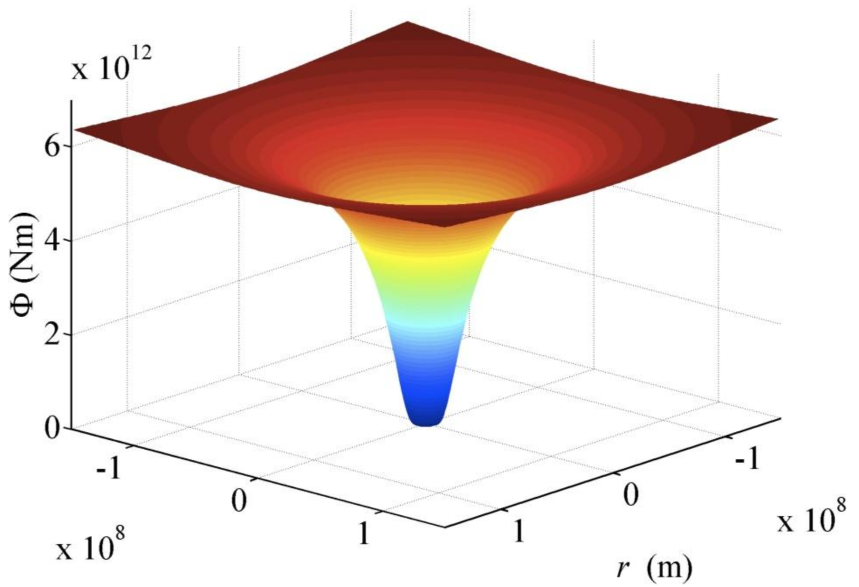
# Most striking properties

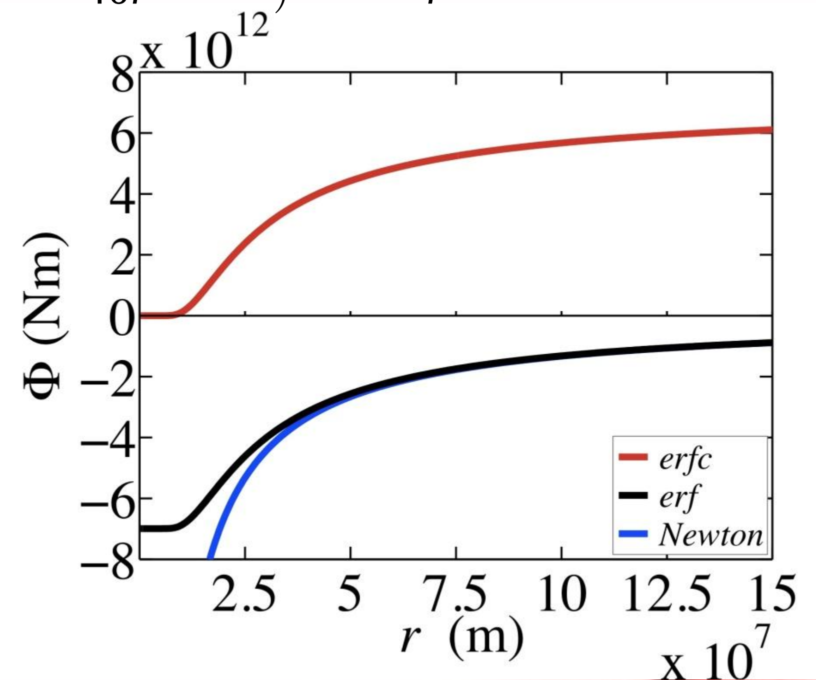
- New set of exact analytical solutions
- Converge towards Einstein's predictions at large distance
- Differs from Einstein's predictions at small distance
- There will be no gravitational collapse in systems described by such a metric
- Black holes without any intrinsic singularity
- Gauge dependent?

# Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{4\pi\sigma^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) = \Phi_{erfc}(r)$$

$$\Phi_{erf}(r) = -\frac{2KMc^4}{4\pi\sigma^3}\left(\frac{1}{r} - \frac{1}{6r^3} + \frac{1}{40r^5} - \dots\right) \cong -\frac{GM}{r}$$



# *erfc* vs *erf* functions

$$erfc\ \ z\ \ = 1 - erf\ \ z$$

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# Two new axisymmetric components

- A rotation term:

$$\omega_{st} = \frac{d\phi}{dt} \Rightarrow +\frac{2K}{\omega_{st}} d\phi dt$$

- An expansion term:

$$v_{st} = \frac{dr}{dt} \Rightarrow +\frac{2Kv_{st}}{c^2}\left[1 - \frac{2K}{c^2} erf\left(\frac{u}{4\pi\sqrt{2}r}\right)\right]^{-1}$$

# Very Brief Pause…

The axisymmetric metric
can be seen as explaining
why any  massive body
in the universe
is rotating
and its associated space-time
looks like expanding.

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# Back to the Central Limit Theorem

When the number of functions that are convolved is not infinite…

A convergence error will emerge.

This error will have three components.

# Central Limit Theorem: convergence error

$$E_r(y) = f(y) - N(y) = \frac{\mu_3}{6\sigma^3\sqrt{n}}\left(\frac{y^3}{\sigma^3} - \frac{3y}{\sigma}\right)N(y) + O\left(\frac{1}{\sqrt{n}}\right)$$

$$\text{Mapping} \Rightarrow \frac{y}{\sigma} \rightarrow \frac{\sqrt{2}x}{\sigma} \Rightarrow \frac{x}{\sigma} = \frac{\sigma}{\sqrt{2}r}$$

$$\left(\frac{y^3}{\sigma^3} - \frac{3y}{\sigma}\right) \Rightarrow \left(\frac{x^3}{2\sqrt{2}\sigma^3} - \frac{3x}{\sqrt{2}\sigma}\right)$$

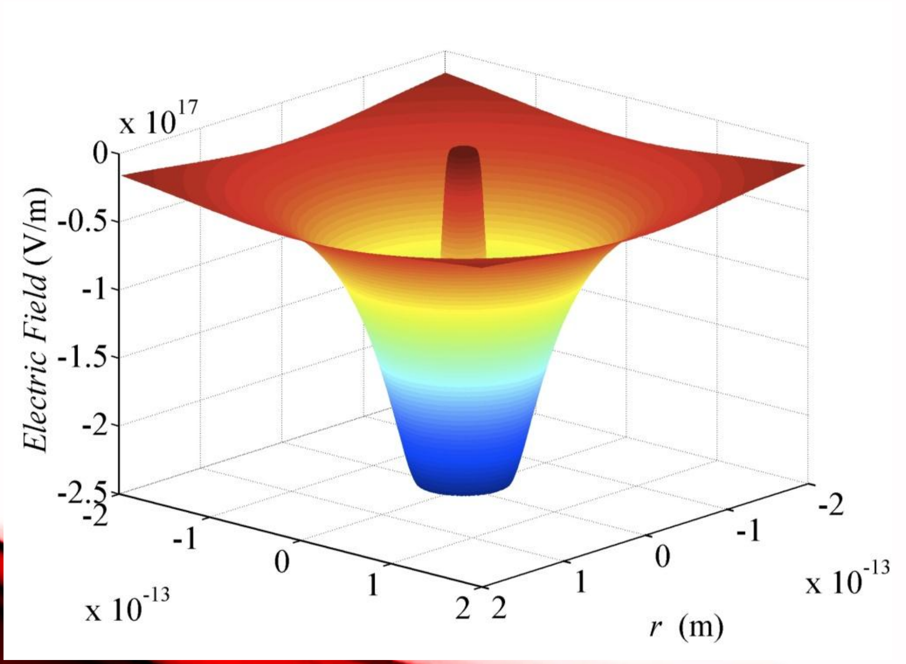
$$\nabla^2\Phi = \frac{2K_\sigma M c^4}{4\pi^{3}\sigma r^5}\left[1 - \frac{\mu_3}{2\sqrt{2n}\sigma^2 r} + \frac{\mu_3}{12\sqrt{2n}r^3}\right]\exp\left(\frac{-\sigma^2}{2r^2}\right)$$

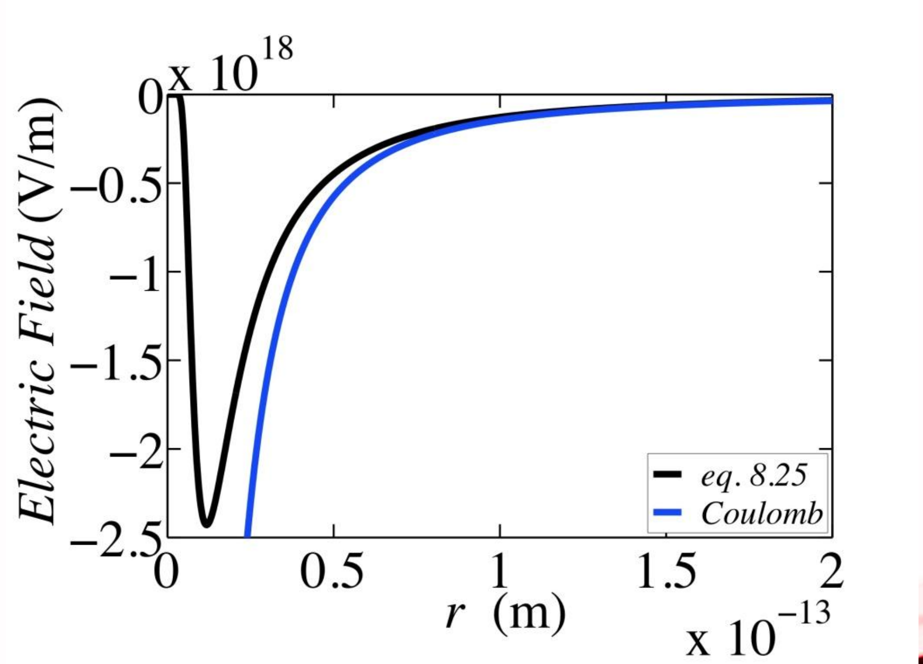
# Emergence of Coulomb's field

$$\mu_3 = \frac{8\pi c_2 Q^2}{m_{ref} u \varepsilon_0^2}$$

$$g_e(r) = \frac{\mu_3 u \varepsilon_0}{32\pi^2 c_2 r^2} erfc\left(\frac{c_3 \varepsilon_0}{64\pi^2 c_2 \sqrt{2} r}\right)$$

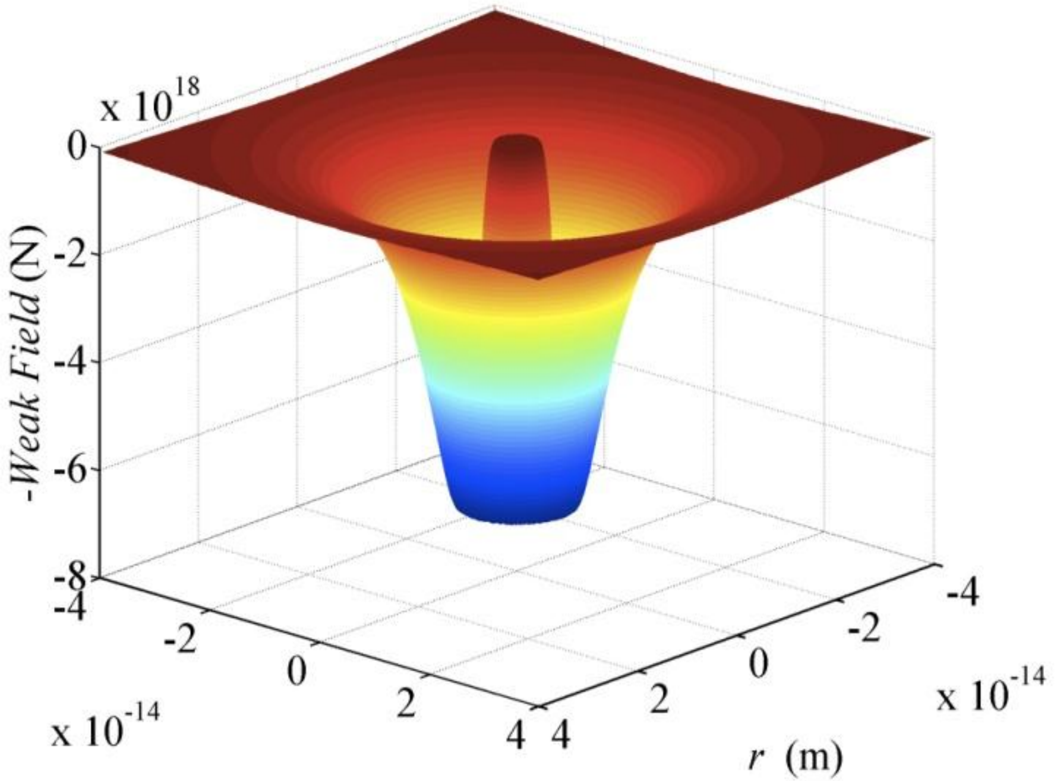
$$F_e(r) = \frac{Q^2}{4\pi\varepsilon_0 r^2} erfc\left(\frac{c_3 \varepsilon_0}{64\pi^2 c_2 \sqrt{2} r}\right)$$



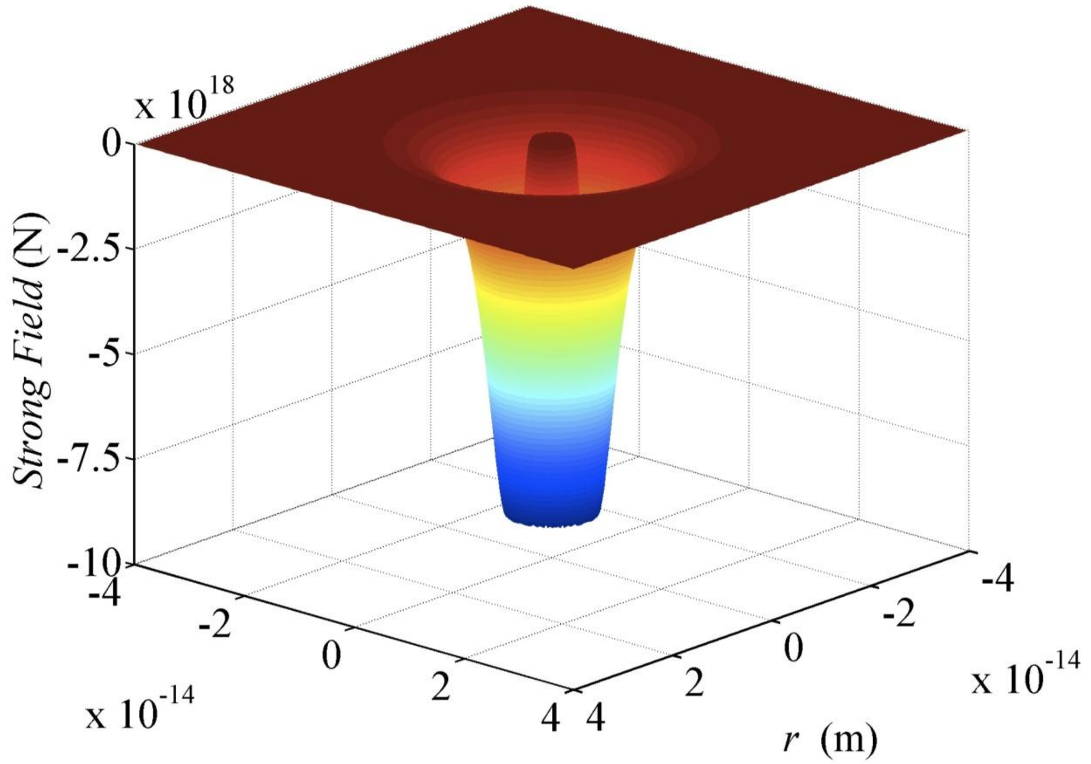
# Emergence of a weak nuclear field

$$F_w(r) = \frac{c_3 Q^2}{128 c_2 \pi^3 \sqrt{2\pi r^3}} \exp\left(-\frac{c_3^2 \varepsilon_0{}^2}{2(64\pi^2)^2 c_2^2 r^2}\right)$$



# Emergence of a strong nuclear field

$$F_s(r) = -\frac{c_3^3 Q^2 \varepsilon_0{}^2}{24 c_2^3 \pi^3 \sqrt{2\pi}(16\pi)^4 r^5} \exp\left(-\frac{c_3^2 \varepsilon_0{}^2}{2(64\pi^2)^2 c_2^2 r^2}\right)$$



# In other words…

The four basic interactive forces of physics can be seen as emergent phenomena described by specific mathematical patterns, when analyzed through the appropriate representation and interpretation schemes

There is nothing such as a free lunch…

# There is much more
# than
# a free lunch!

# Predicting the values of the fundamental constants from various mappings

$$G = \frac{2Ku^2}{c\delta\tau^5}$$

$$\hbar = \frac{c_1}{9\sigma^3\sqrt{N_a}u}$$

$$\frac{\varepsilon_0}{32\pi^2 c_2} = \frac{9GM\hbar\sqrt{\pi}}{8c_1}$$

$$2\sigma_{warp} = \frac{c_3\varepsilon_0}{32c_2\pi^2} = \frac{u_{warp}}{2\pi}$$

$$Q^2 = \frac{(\Delta\mathrm{amu})c^2c_3\varepsilon_0^2}{18c_2\pi^3}$$

# Predicting the values of the fundamental constants from various mappings

$$k = \frac{300\rho_i}{2N_a m_{H_2O}\ 1\ \mathrm{dof/m^3}}\left[\frac{1\ \mathrm{J}}{1\ \mathrm{K}}\right]$$

$$m_e = 0.1 m_{H_2O}\frac{1kg}{M_{Sun}\exp\left[-1/16\pi^2\right]}$$

$$m_p = 0.1\kappa_{\min\max} m_{ref\,\min} = \frac{0.01\ \mathrm{kg}^2}{M_{Earth}}$$

$$N_a \cong \frac{M_{Earth}}{10\exp\left[-1/16\pi^2\right]\ \mathrm{kg}}$$

# In other words…

Once a coherent set of physical units is defined, the values of the fundamental constants of Nature can be seen as numerical parametric patterns that can be predicted!

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# I Love the Central Limit Theorem!!!

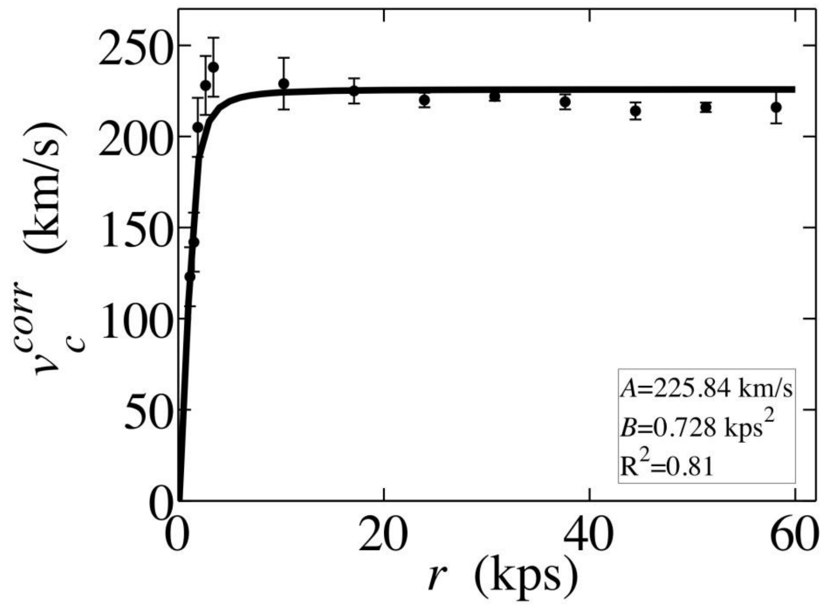
- The Central Limit Theorem is an intrinsic property of the convolution of a large number of positive definite functions.
- It can be used the describe a Gaussian star.
- The convolution of a Gaussian is a Gaussian.
- The convolution of a large number of Gaussian stars will converges toward a Gaussian galaxy.
- The convolution of a large number of Gaussian galaxies will converges toward a Gaussian Universe.

# Very Very Brief Pause…

Dark matter
might not be necessary
to explain
the orbital rotation velocity
at and beyond
the visible outer edge
of distant galaxies

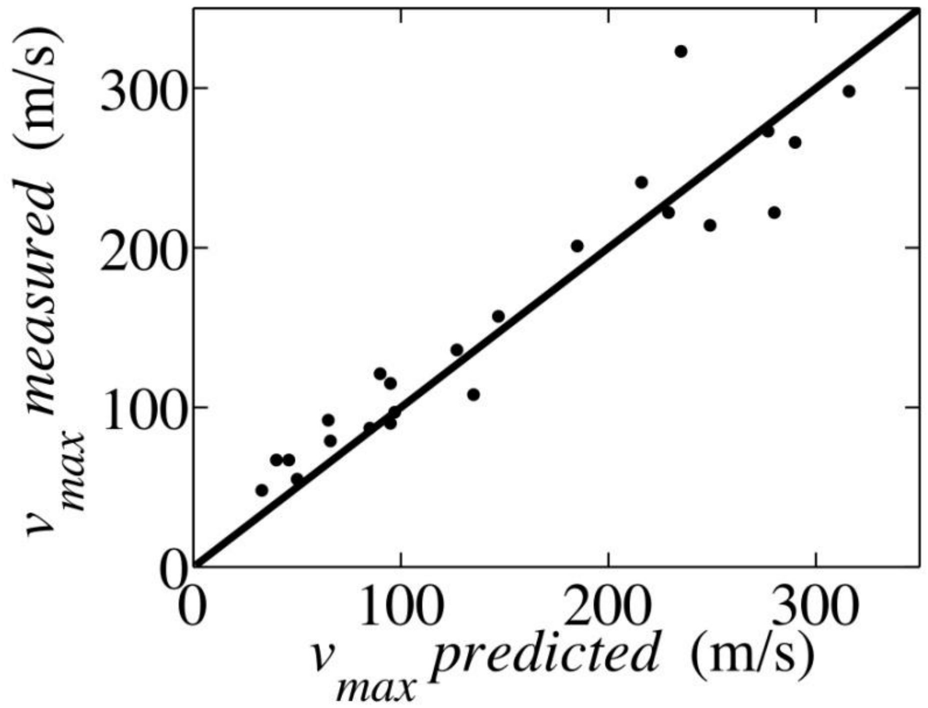
# Dark Matter: rotational velocity of galaxies

$$v_{\exp} = A\exp\left(\frac{-B}{r^2}\right)$$




Predicted and measured values of the orbital rotation velocity of the NGC 801 galaxy as a function of its radius.
The experimental data are from Table 4, chapter 7 of Broeils (1992)

# Dark Matter: rotational velocity of galaxies




Predicted vs measured values
of the orbital rotation velocity of some distant galaxies.
from table 1 and table 4 of Blok and McGaugh (1997)

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# Physicist Take Home Messages

- According to the present paradigm, the four physical interactions are not empirical laws. They can be seen as emerging phenomena when pattern recognition techniques are used to describe a space-time manifold.
- The fundamental constants of nature can be linked to the unique intrinsic and emergent feature of the model $\sigma$ and their values can be interpreted as numerical patterns that can be predicted.

# Physicist Take Home Messages

- The model can be applied to some stars, some galaxies and…

  to the whole Universe.

- The Central Limit Theorem is one of the basic tool to study the Signature of the Universe.

# Pattern Recognition Specialists
# Take Home Message

PATTERN RECOGNITION MIGHT BE THE MOST FUNDAMENTAL SCIENCE!

# Document Recognition Specialist
# Take Home Message

SOME DOCUMENT ANALYSIS TECHNIQUES

CAN BE APPLIED TO STUDY THE BOOK OF NATURE

AS WELL AS

THE ENCYCLOPEDIA OF THE UNIVERSE!

# Signature Verification Specialist
# Take Home Message

THE STUDY OF THE UNIVERSE

IS A VERY THOUGH

SIGNATURE VERIFICATION PROBLEM

ONE SUBJECT…

ONE SPECIMEN…

# To investigate further…